Please have out...

- notebook
- calculator

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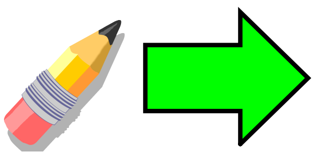
6.3 How can we find probabilities when each observation has two possible outcomes?



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What are we learning today?

John Doe claims to possess ESP.

An experiment is conducted:

- A person in one room picks one of the integers 1, 2, 3, 4, or 5 at random.

- In another room, John Doe identifies the number he believes was picked.

-Three trials are performed for the experiment.

-John got the correct answer twice. What are the chances of this?

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Binomial Distribution

Each observation is binary: it has one of two possible outcomes.

Examples:

- Heads or tails on a coin toss.
- Accept, or decline an offer from a bank for a credit card.
- Have, or do not have, health insurance.
- Vote yes or no on a referendum.
- Vote McCain or Obama for President

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Conditions for the Binomial Distribution

Each of n trials has two possible outcomes: "success" or "failure"

Each trial has the same probability of success, denoted p.

The n trials are independent.

The binomial random variable X is the number of successes in the n trials.

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Try It!

If this is a success, then what is the failure?

Getting an A in stats

Having 2 girls

Passing a class

Buying a diamond engagement ring

Favorite color is pink

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Remember?

First we need to remember...

Tell the person beside you how to find 7! (factorial)

7 x 6 x 5 x 4 x 3 x 2 x 1

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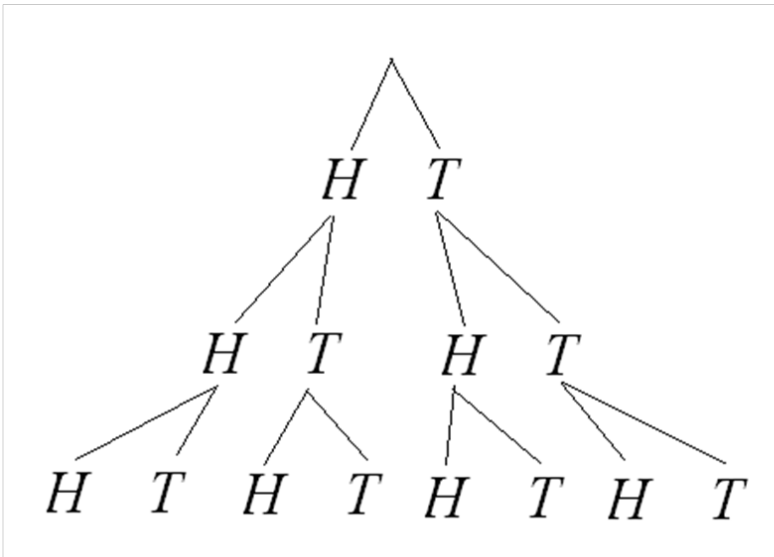
Ex/ Coin Toss

Make a tree diagram for tossing a coin 3 times



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Is this binomial?

- Are there only 2 outcomes (success or failure)?

- Is the probability of success the same for each trial?

- Are the trials independent?
    - Yes! Tails or Heads = 2 outcomes

    - Yes! .5 chance for heads or tails

    - Yes! Flipping a coin is an independent event

    Sooooo, Yes! Binomial

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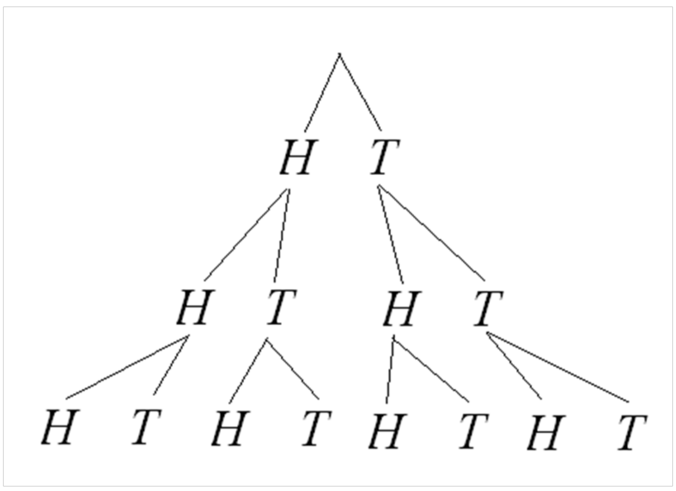
What's the probability of our coin landing on tails exactly twice?

- How many ways are there to do this?

- What's the probability of each one?



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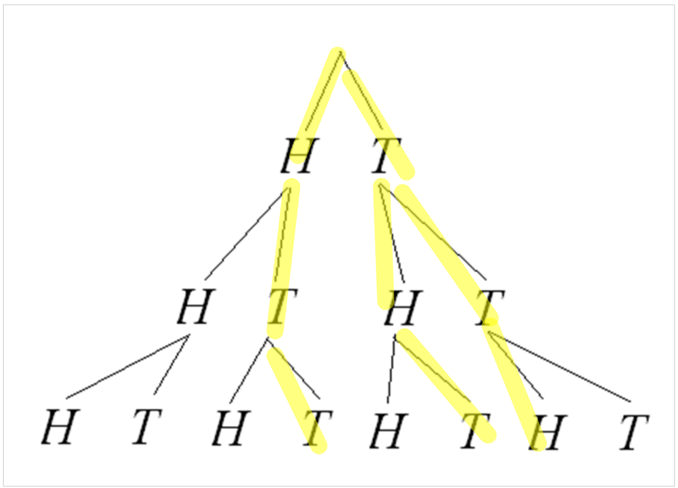

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Probabilities for a Binomial Distribution

Denote the probability of success on a trial by p.

For n independent trials, the probability of x success equals:

$$\mathrm{P(x)} = \frac{\mathrm{n!}}{\mathrm{x!(n-x)!}} p^x (1-p)^{n-x}, \quad x = 0, 1, 2, \ldots, n$$

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Always check for binomial first!

Before using the binomial distribution, check that the 3 conditions apply:

binary data (success or failure) 1

the same probability of success for each trial (p) 2

independent trials 3

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Back to John Doe

John Doe claims to possess ESP.

An experiment is conducted:

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Is John Doe's Case Binomial?

Are there only two outcomes?

Does each trial have the same probability of success?

Are the trials independent?

- yes - guess # right or guess # wrong

- yes - 1/5 = .2

- yes

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It's binomial! Yay!

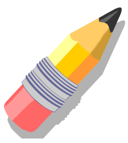
What is n? (How many trials are we performing?)

What is p? (What is the probability of success?)

What is x? (How many successes are we looking for?)

$n = 3$

$p = 1/5 = .2$



$x = 2$

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John Doe

If John Doe does not actually have ESP and is just guessing the number, what is the probability that he'd make a correct guess on two of the three trials?

the three ways John Doe could make two correct guesses in the three trials are: SSF, SFS, and FSS.

each of these has probability:
$(.02)^2 (.08) = .032$

the total probability of two correct guesses is:
$3(.032) = .096$

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Plug it in!

n = 3 x = 2 p = .2

$$P(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}, \quad x = 0, 1, 2, \ldots, n$$

$$P(2) = \frac{3!}{2!1!}(0.2)^2(0.8)^1 = 3(0.04)(0.8) = 0.096$$

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Try It!

1000 employees , 50% Female

None of the 10 employees chosen for management training were female

Is it binommial?

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Binomial?

Binary - Female or Male 1

If employees are selected randomly, the probability of selecting a female is .5 and the probability of selecting a male is .5 2

With random sampling of 10 employees from a large population, the outcome for one trial does not depend on the outcome for another trial 3

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It's binomial! Yay!

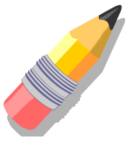
What is n? (How many trials are we performing?)

What is p? (What is the probability of success?)

What is x? (How many successes are we looking for?)

n = 10

p = .5



x = 0

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Plug it in!

n = 10 p = .5 x = 0

$$P(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}, \quad x = 0, 1, 2, \ldots, n$$

$$P(0) = \frac{10!}{0!10!} (0.50)^0 (0.50)^{10} = 0.001$$

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Back to the female employees...

1000 employees, 50% females
None of the 10 chosen for training were female

We found: $$P(0) = \frac{10!}{0!10!}(0.50)^0(0.50)^{10} = 0.001$$

mean: $10(.5) = 5$

st. dev. : $\sqrt{10 \cdot .5(1-.5)} = 1.58$

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Where we'll pick up tomorrow...

- more binomial distribution
- shortcuts (woohooo)

P.S. it'll be the last day of notes in this class EVER!!

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BELLWORK

normcdf
invNorm

The average number of pancakes an OC student can eat is 4 with a sd of 0.68.

What is the probability a student can eat more than 6 pancakes?

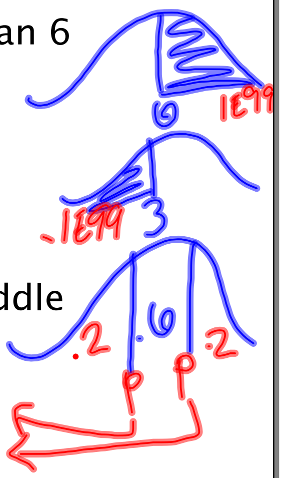
normcdf(6, 1E99, 4, .68) = .002

What is the probability a student eats less than 3 pancakes?

normcdf(-1E99, 3, 4, .68) = .07

What amounts of pancakes correspond to the middle 60%?

invNorm(.2, 4, .68) = $\boxed{3.43 \text{ to } 4.57}$
invNorm(.8, 4, .68) =



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6.3 Continued...

Calculators OUT!

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Main focus of yesterday...

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Conditions for a binomial dist.

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Probabilities for a binomial dist.

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Binomial dist. Mean and St. Dev.

The binomial probability distribution for n trials with probability p of success on each trial has mean μ and standard deviation σ given by:

$$\boxed{\mu = np,} \quad \boxed{\sigma = \sqrt{np(1-p)}}$$

mean St. dev.

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Practice Example

Data: n
- 262 police car stops in Philadelphia in 1997.
x 207 of the drivers stopped were African-American.
- In 1997, Philadelphia's population was 42.2% African-American. p(.422)

Does the number of African-Americans stopped suggest possible bias, being higher than we would expect (other things being equal, such as the rate of violating traffic laws)?

In other words, within what interval does the majority of the data fall? Can I use the mean and standard deviation to help me with this?

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WAIT!

I need to make sure it's binomial and identify my n and p.

What is success and what is failure?

Does each trial have the same probability of success? What is it?

Are the trials independent?

What is my n? (How many trials am I doing?)

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Back to the Example...

Assume:
262 car stops represent n = 262 trials.

Successive police car stops are independent.

P(driver is African-American) is p = 0.422.

Calculate the mean and standard deviation of this binomial distribution:

$\mu = np = 262(.422) = \boxed{111}$

$\sigma = \sqrt{111(1-.422)} = \boxed{8}$

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Back to the Example...

Recall: Empirical Rule

When a distribution is bell-shaped, close to 100% of the observations fall within 3 standard deviations of the mean. Draw the curve.

$$\mu - 3\sigma = 111 - 3(8) = 87$$

$$\mu + 3\sigma = 111 + 3(8) = 135$$

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Binomial Distribution

Is there a number of successes and failures that we are looking for?

How many successes do I need to say that my binomial distribution is doing a good job?

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Binomial Distribution

The binomial distribution can be well approximated by the normal distribution when the expected number of successes, np, and the expected number of failures, n(1−p) are both at least 15.

$$\boxed{\begin{array}{l} np = 15^{+} \\ n(1-p) = 15^{+} \end{array}}$$

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Back to John Doe

Is correctly guessing 2 of 3 trials enough to prove to me that he has ESP?

x n

p=.2

$np = 3(.2) = .6$ ☹ $n(1-p) = 3(.8) = 2.4$ ☹

These should both be 15!

How many trials where he can guess 2 out of 3 would be enough for you to believe? 100? 1,000? 10,000?

$np = 100(.2) = 20$ ☺ $n(1-p) = 100(.8) = 80$ ☺

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CALCULATOR COMMANDS

WOOOHOOOOO! Our favorite part!

2nd VARS (like before)
A 0: binompdf (NOT cdf...yikes)
Type in (n, p, x)

$\boxed{\text{binompdf(n,p,x)}}$ n=trials

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John Doe - Calc Style

The probability of exactly 2 correct guesses is the binomial probability with n = 3 trials, x = 2 correct guesses and p = 0.2 probability of a correct guess.

binompdf(3, .2, 2) = $\boxed{.096}$

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Sexist Boss - Calc Style

1000 employees, 50% Female

None of the 10 employees chosen for management training were female.

n = 10
p = .5
x = 0

binompdf(10, .5, 0)

= .00098

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Try It!

I have 1000 jelly beans in a jar. 30% of them are watermelon. I reach in and grab 10 jelly beans. What is the chance that exactly 2 of them are watermelon flavored?

$n=10$
$p=.3$
$X=2$

binompdf(10, .3, 2)

$= \boxed{.23}$

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Try It!

I am awesome at archery. I can hit a bulls-eye with 67% accuracy. What's the chance that I make 5 out of 7 shots at the bulls-eye?

$n=7$
$p=.67$
$x=5$

binompdf(7, .67, 5)

$= \boxed{.31}$

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Just My Imagination...

Create your own binomial pdf or normal cdf problem
& solve it!

I will be using some tomorrow ☺

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Homework!

6.3 Page 299
#33b, 36, 38, 39, 40, 41, 43,8 a and b

OMGosh! Last homework assignment of the year!
Bring back your stat books NOW!

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