BELL shaped curve WORK

The average test score was a 70, with a standard deviation of 10.

1. What proportion of people scored below a 60%?

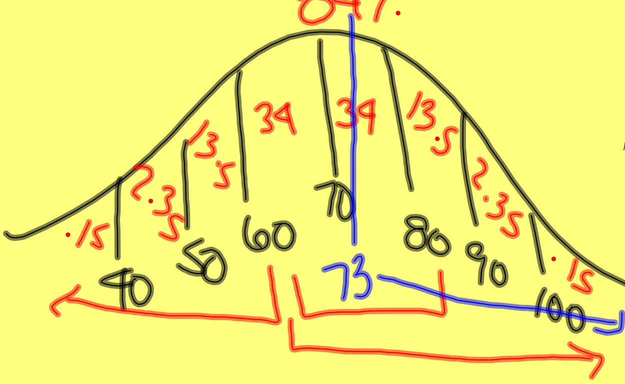
16%

2. What proportion of people scored between a 60% and 80%?

68%

3. What proportion of people passed the test?

84%.



Nov 28-2:24 PM

YESTERDAY...

- We used the Empirical Rule to find probabilities for areas under a normal curve.
- If I asked you the probability that a normal random variable takes a value less than 1 standard deviation above the mean, you should say…
- 84% (Good job! ☺)
- What if I asked you the probability that a normal random variable takes a value less than 1.43 standard deviations above the mean?

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WAIT!!

- 1.43 standard deviations…that's not part of the Empirical Rule!

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Standard Normal Probabilities

z-scores can be used to calculate the probabilities of a normal random variable using the **normal tables** in the back of the book

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Standard Normal Probabilities

Table A enables us to find normal probabilities

- It tabulates the normal cumulative probabilities falling below the point μ+zσ

To use the table:

1. Find the corresponding z-score
2. Look up the closest standardized score (z) in the table.
3. First column gives z to the first decimal place
4. First row gives the second decimal place of z
5. The corresponding probability found in the body of the table gives the probability of falling below the z-score

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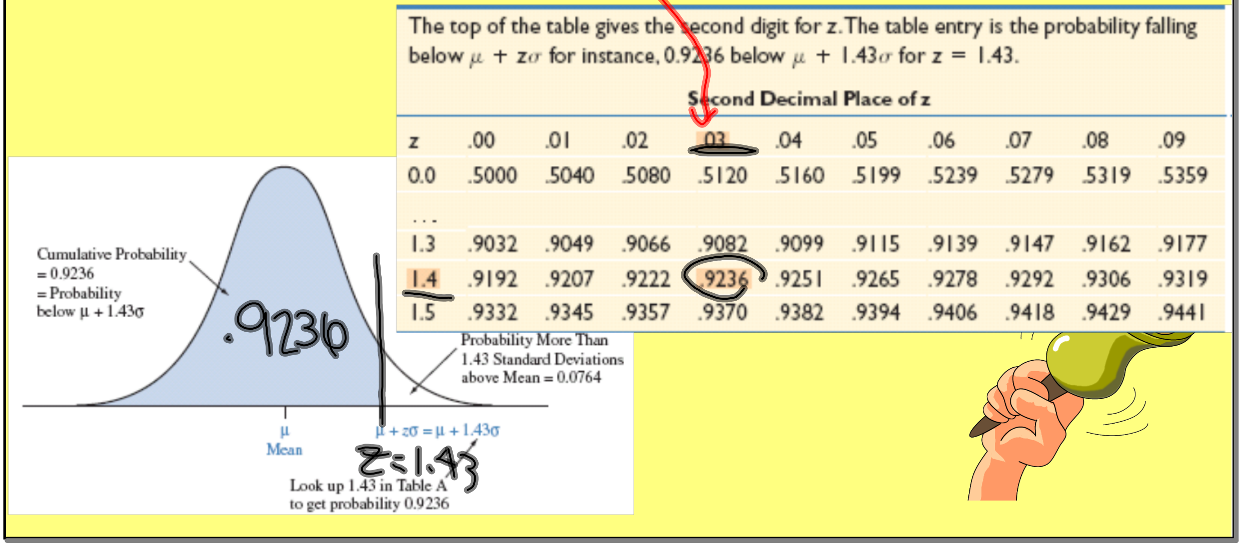
Example

Find the probability that a normal random variable takes value less than 1.43 standard deviations above μ

- P(z<1.43)=.9236



The top of the table gives the second digit for z. The table entry is the probability falling below μ + zσ for instance, 0.9236 below μ + 1.43σ for z = 1.43.

Second Decimal Place of z

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
. . .										
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9139	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9278	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441

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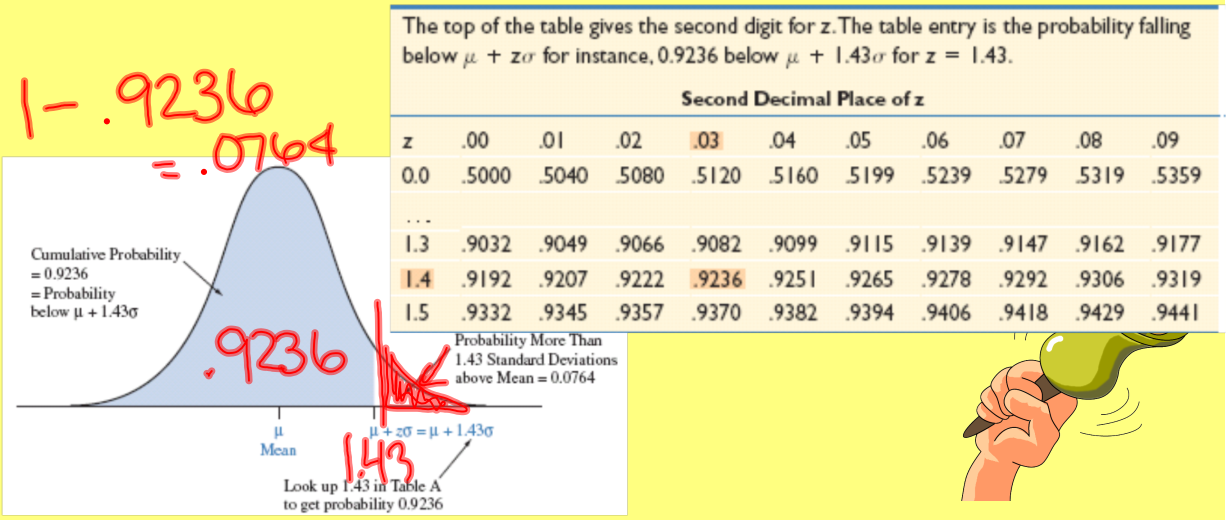
Example

Find the probability that a normal random variable takes a value greater than 1.43 standard deviations above μ:

- P(z>1.43)=1−.9236=.0764



The top of the table gives the second digit for z. The table entry is the probability falling below $\mu + z\sigma$ for instance, 0.9236 below $\mu + 1.43\sigma$ for z = 1.43.

Second Decimal Place of z										
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
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1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441

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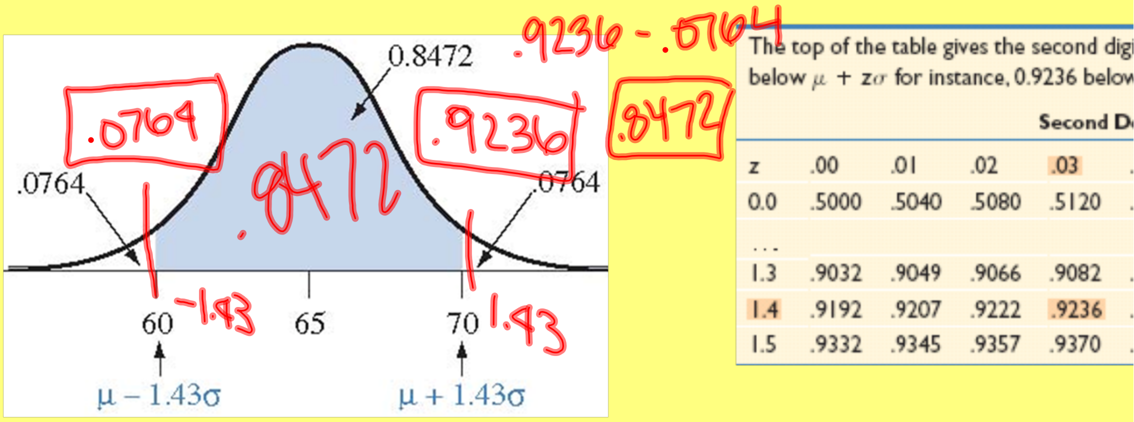
Example

Find the probability that a normal random variable assumes a value within 1.43 standard deviations of μ on either side

- Probability below 1.43σ = .9236
- Probability below −1.43σ = .0764 (1−.9236)
- P(−1.43<z<1.43) =.9236−.0764=.8472



The top of the table gives the second dig
below $\mu + z\sigma$ for instance, 0.9236 below

Second D				
z	.00	.01	.02	.03
0.0	.5000	.5040	.5080	.5120
. . .				
1.3	.9032	.9049	.9066	.9082
1.4	.9192	.9207	.9222	.9236
1.5	.9332	.9345	.9357	.9370

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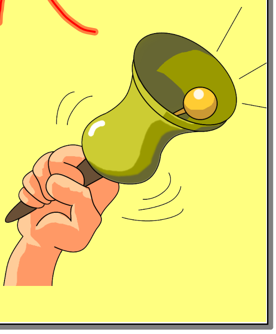
Remember this ...

Z-scores can be used to compare observations from different normal distributions

Example:
You score 650 on the SAT which has $\mu=500$ and $\sigma=100$ and 30 on the ACT which has $\mu=21.0$ and $\sigma=4.7$. On which test did you perform better? Compare z-scores

SAT: 1.5 → .9332

ACT: 1.91 → .9719



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Remember this ...

SAT:

$$z=\frac{650-500}{100}=1.5$$

ACT:

$$z=\frac{30-21}{4.7}=1.91$$

Since your z-score is greater for the ACT, you performed better on this exam

In what percentile did you score for each exam?

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Remember this ...

Find probability to the left of −1.64

.0505

Find probability to the right of 1.56

1 – .9406 = .0594

Find probability between −.50 and 2.25

.9878 – .3085 = .6793

↓ ↓

.3085 .9878

Nov 28-2:24 PM

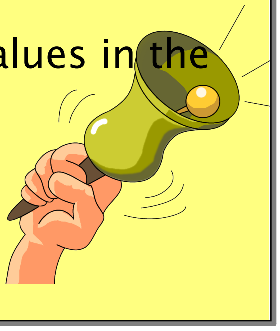
How Can We Find the Value of z for a Certain Cumulative Probability?

To solve some of our problems, we will need to find the value of z that corresponds to a certain normal cumulative probability

To do so, we use Table A in reverse:

Rather than finding z using the first column, find the probability in the body of the table

The z-score is given by the corresponding values in the first column and row



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How Can We Find the Value of z for a Certain Cumulative Probability?

Example: Find the value of z for a cumulative probability of 0.025? 0

Look up the cumulative probability of 0.025 in the body of Table A.

-1.96

A cumulative probability of 0.025 corresponds to z = −1.96.

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PRACTICE EXAMPLE

Find the value of z for a cumulative probability of 0.975.

z = 1.96

Z=1.96

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PRACTICE EXAMPLE

The probability that a standard normal random variable assumes a value that is < z is 0.975. What is z?

z=1.96

The probability that a standard normal random variable assumes a value that is > z is 0.025.

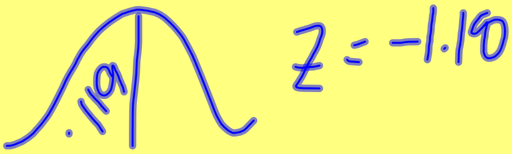
z=1.96

The probability that a standard normal random variable assumes a value that is > z is 0.881.

Look up .119; z=−1.18

The probability that a standard normal random variable assumes a value that is < z is 0.119.

z= −1.18



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What if I asked...

Adult systolic blood pressure is normally distributed with μ = 120 and σ = 20. What percentage of adults have systolic blood pressure less than 100?

$$z = \frac{100 - 120}{20} = \frac{-20}{20} = \boxed{-1}$$

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What if I asked...

Adult systolic blood pressure is normally distributed with μ = 120 and σ = 20. What percentage of adults have systolic blood pressure greater than 100?

$1-.1587= \boxed{.8413}$

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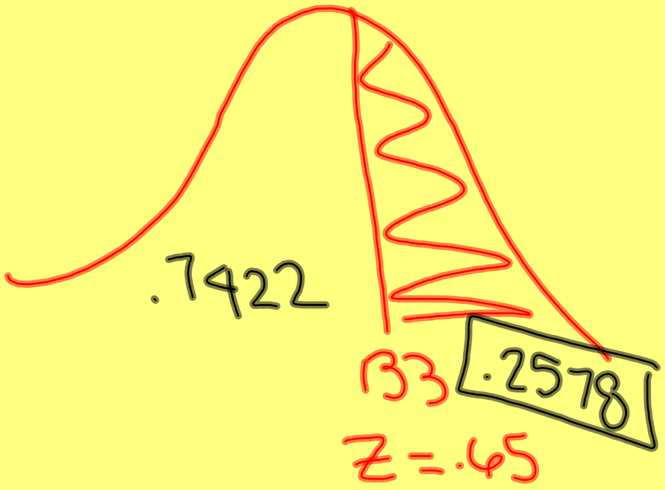
What if I asked...

Adult systolic blood pressure is normally distributed with μ = 120 and σ = 20. What percentage of adults have systolic blood pressure greater than 133?



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What if I asked...

Adult systolic blood pressure is normally distributed with μ = 120 and σ = 20. What percentage of adults have systolic blood pressure between 100 and 133?

.7422 - .1587

$= \boxed{.5835}$

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What if I asked...

Adult systolic blood pressure is normally distributed with μ = 120 and σ = 20. What is the 1st quartile?

$$z = \frac{x-\mu}{\sigma}$$

$$20 \cdot -.67 = \frac{x-120}{\cancel{20}} \cancel{20}$$

$$-13.4 = x - 120$$

$$+120 \quad +120$$

$$\boxed{106.6}$$

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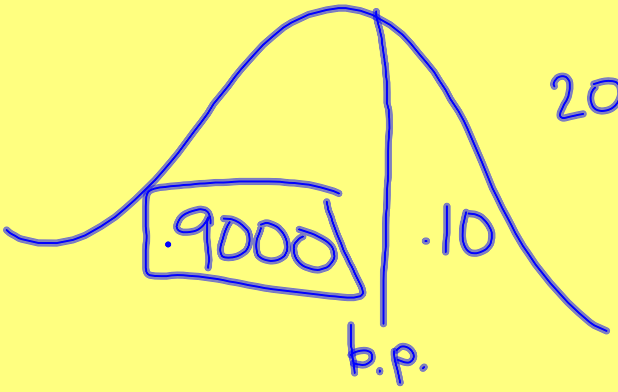
What if I asked...

Adult systolic blood pressure is normally distributed with μ = 120 and σ = 20. 10% of adults have systolic blood pressure above what level?



z = 1.28

$$20 \cdot 1.28 = \frac{X-120}{\cancel{20}} \cdot \cancel{20}$$

$$25.6 = X \qquad -20$$

$$+120 \qquad +120$$

$$X = \boxed{145.6}$$

Nov 28-2:24 PM