Bellwork!

Which is the better bet to take?

You bet $5 and...
- win $20 if a 1 or 3 are rolled
- lose $10 if any other number is rolled

x	P(x)
$20 ×	.33
-$10 ×	.67

$6.60
-$6.70
-.10

You bet $5 and...
- win $15 if a 1 or 3 or 5 are rolled
- lose $10 if any other number is rolled

x	P(x)
15	.5
-10	.5

$7.50
- $5.00
$2.50 *

Nov 28-2:24 PM

WELCOME!

<u>You'll Need Out:</u>
- Notes
- Calculator

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TRY IT!

- Grade Distribution Worksheet!!
- 5 minutes for a –g!

a.

x	P(x)
4	
3	
2	
1	
0	

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a.

x	P(x)
4	.15 =
3	.35 =
2	.30 =
1	.16 =
0	.04 –

b. yes

$0 \le P(x) \le 1$ ✓

$.15 + .35 + .30 + .16 + .04 = 1$ ✓

c. .5

$.3 + .16 + .04$

d. .85

e. 15 = A B = 35
C = 30 D = 16
F = 4

f. Value of X

g. 2.41

$4(.15) + (3)(.35) + 2(.3) + 1(.16) + 0(.04) =$

Dec 4-11:04 AM

a) $P(4) = .15$
$P(3) = .35$
$P(2) = .3$
$P(1) = .16$
$P(0) = .04$

b) $0 \leq P(x) \leq 1$
$\Sigma P(x) = 1$

c) $.3 + .16 + .04 = .5$

d) $.5 + .35 = .85$

e) A - 15
B - 35
C- 30
D - 16
F - 4

f) $.15(4) + .35(3) + .3(2) + .16(1) + .04(0) = 2.41$

g) value

Nov 30-6:54 AM

TRY IT!

A wheat farmer in Canada finds that his annual profit is $80,000 if the summer weather is typical, $50,000 if the weather is unusually dry, and $20,000 if there is a severe storm that destroys much of his crop. Weather bureau records indicate that the probability is 0.7 of typical weather, 0.2 of unusually dry weather, and 0.10 of a severe storm. In the next year, let X be the farmer's profit.

a) Construct the probability distribution of X.

b) What is the probability that the profit is $50,000 or less?

c) Find the mean of the probability distribution of X.

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6.2

Probabilities for Bell-Shaped Distributions

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Learning Objectives

- Normal Distribution
- 68-95-99.7 Rule for normal distributions
- Z-Scores and the Standard Normal Distribution
- The Standard Normal Table: Finding Probabilities
- Using the TI-calculator: find probabilities
- Using the Standard Normal Table in Reverse
- Using the TI-calculator: find z-scores
- Probabilities for Normally Distributed Random Variables
- Percentiles for Normally Distributed Random Variables
- Using Z-scores to Compare Distributions

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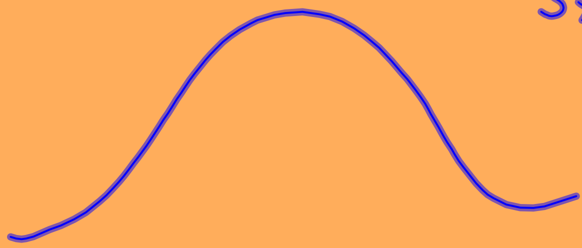
Normal Distribution

The normal distribution is symmetric, bell-shaped and characterized by its mean μ and standard deviation σ.

$\bar{x}$ S_x



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Normal Distribution

The normal distribution is the most important distribution in statistics:

Many distributions have an approximately normal distribution (like height and weight, often test grades)

It approximates many discrete distributions well when there are a large number of possible outcomes

Many statistical methods use "statistical techniques" for normal distributions even when the data are not perfectly bell shaped

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Normal Distribution

- Normal distributions are:
  - Bell shaped
  - Symmetric around the mean

- The mean μ and the standard deviation σ completely describe the density curve
  - Increasing/decreasing μ moves the curve along the horizontal axis
  - Increasing/decreasing σ controls the spread of the curve

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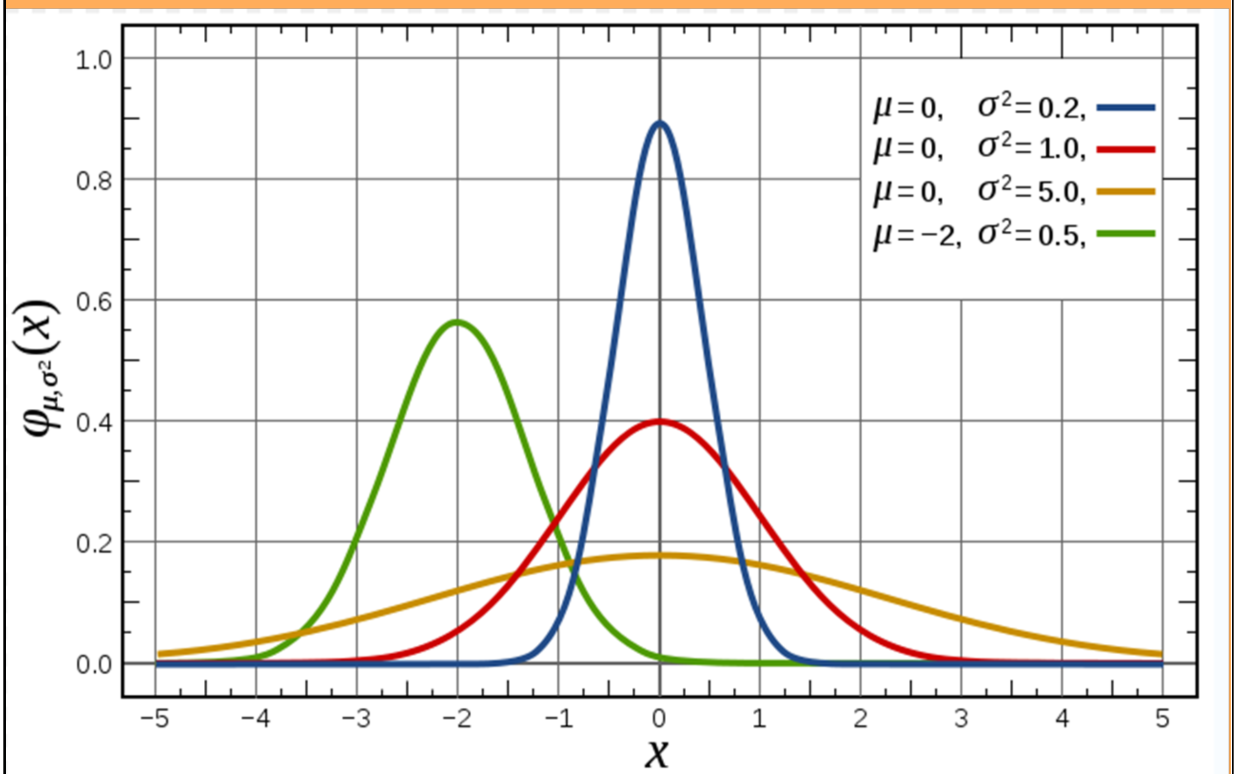

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Try It!

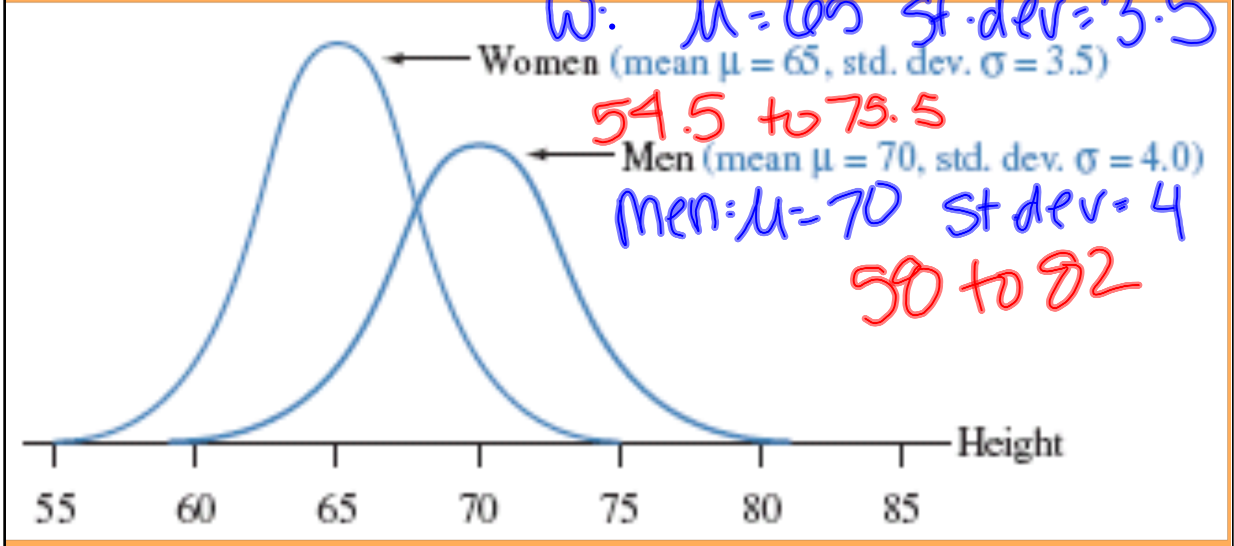
Within what interval do almost all of the men's heights fall? Women's heights? (Empirical Rule)

W: μ = 65 st.dev = 3.5

54.5 to 75.5

men: μ = 70 st dev = 4

58 to 82



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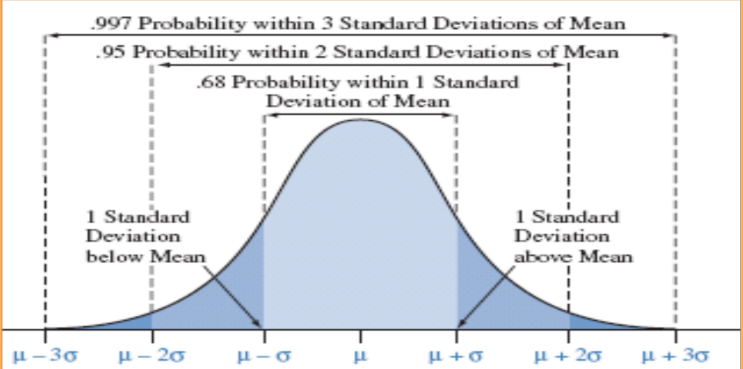
Empirical Rule

- 68% of the observations fall within one standard deviation of the mean
- 95% of the observations fall within two standard deviations of the mean
- 99.7% of the observations fall within three standard deviations of the mean

68%

95%

99.7%



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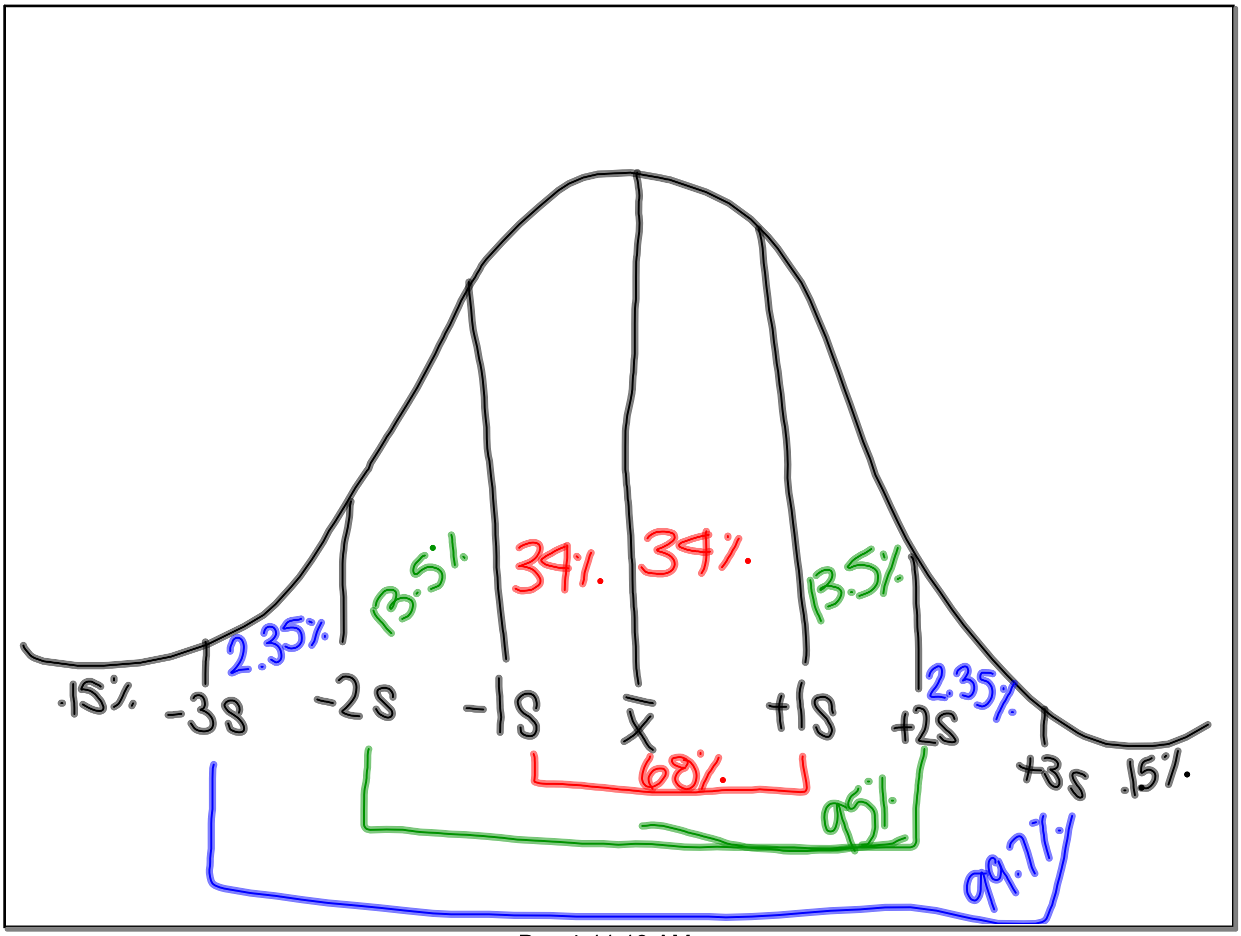

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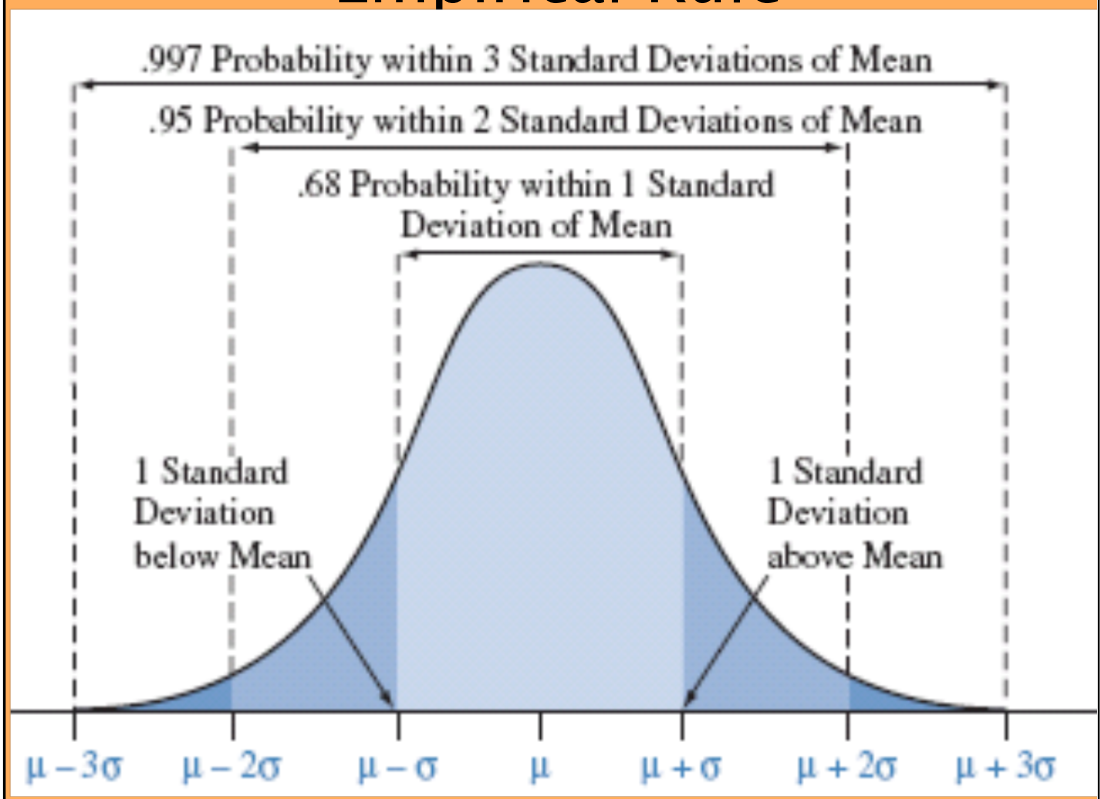
Empirical Rule



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We Do

Heights of adult women

- can be approximated by a normal distribution

  $\mu = 65$ inches; $\sigma = 3.5$ inches

- use the Empirical Rule to approximate the distribution

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You Do

Men? $\mu = 70$ $\sigma = 4.0$

- Within what interval do almost all of men's heights fall?

  58 - 82 (99.7%)

- Within what interval do 68% of men's heights fall?

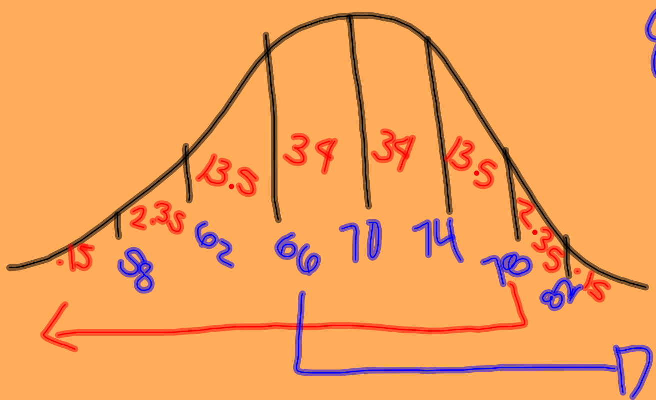
  66 - 74

- What proportion of men are shorter than 78 inches?

  97.5%

- What proportion of men are taller than 66 inches?

  84%



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Z-Scores

The z-score for a value x of a random variable is the number of standard deviations that x falls from the mean

obs. mean

$$z = \frac{x - \mu}{\sigma}$$

st. dev.

A negative (positive) z-score indicates that the value is below (above) the mean

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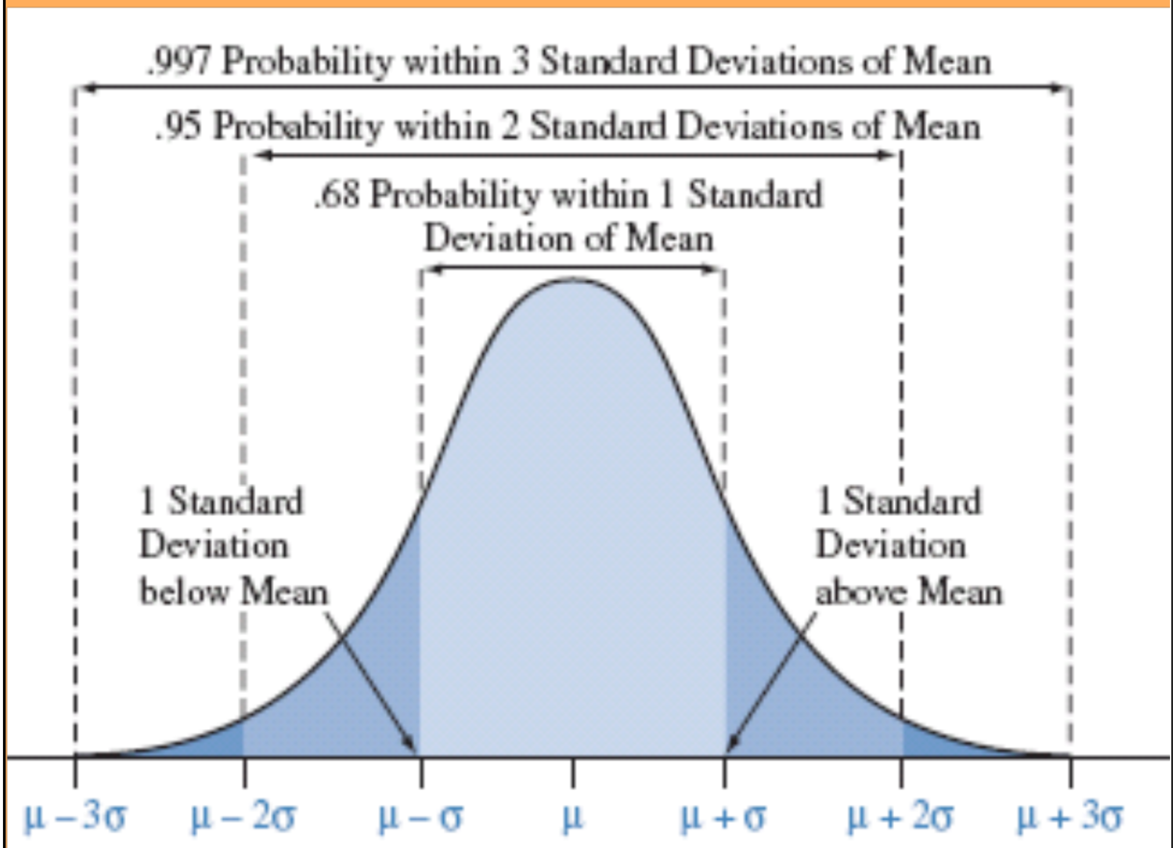

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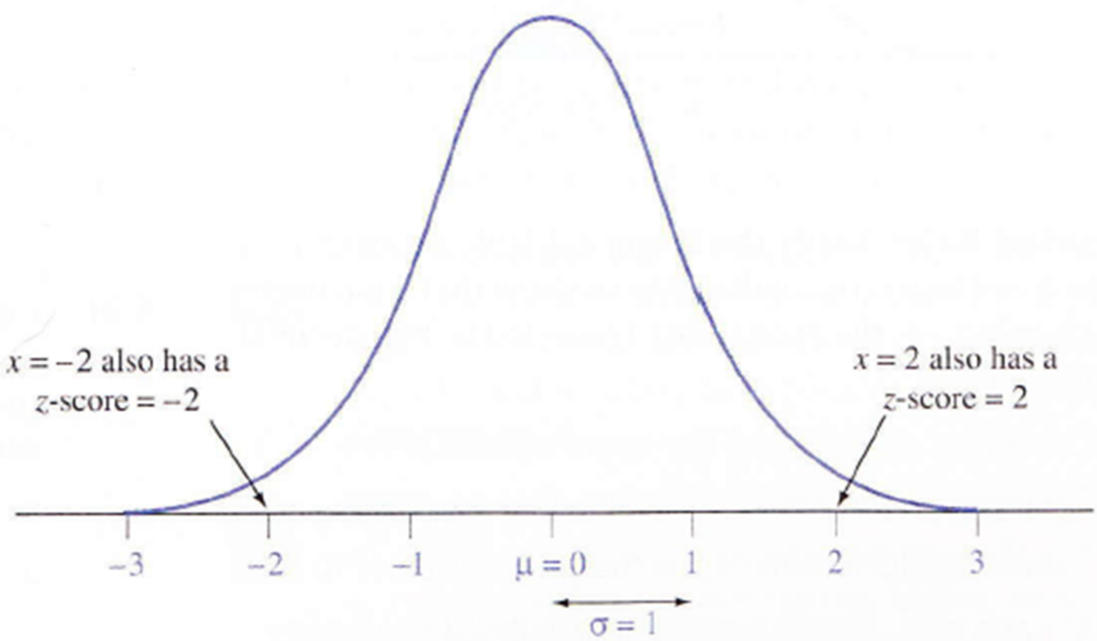
$$z = \frac{x - \mu}{\sigma} = \frac{x - 0}{1} = x.$$



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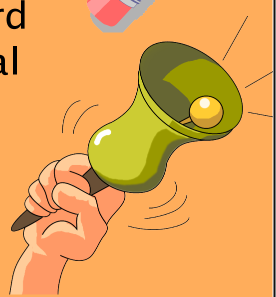
Standard Normal Distribution

A standard normal distribution has mean μ=0 and standard deviation σ=1

When a random variable has a normal distribution and its values are converted to z-scores by subtracting the mean and dividing by the standard deviation, the z-scores have the standard normal distribution.



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Using Z-scores to Compare Distributions

Z-scores can be used to compare observations from different normal distributions
(You CAN compare apples to oranges!! haha)

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YOU DO

You score 650 on the SAT which has $\mu=500$ and $\sigma=100$ and 30 on the ACT which has $\mu=21.0$ and $\sigma=4.7$.

On which test did you perform better? Compare the z-scores!

SAT: $\frac{650-500}{100}$ $z=1.5$

ACT: $\frac{30-21}{4.7}$ $z=1.91$ *

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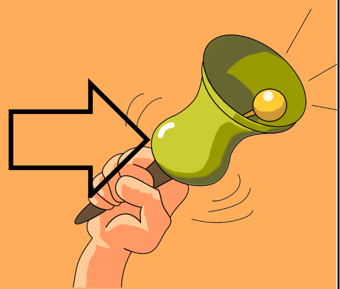
Using Z-scores to Compare Distributions
EXAMPLE

$$z = \frac{650 - 500}{100} = 1.5$$: SAT $\mu = 500$ $\sigma = 100$

$$z = \frac{30 - 21}{4.7} = 1.91$$: ACT $\mu = 21.0$ $\sigma = 4.7$

Since your z-score is greater for the ACT,
you performed better on this exam



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Ticket Out

On your History test you score a 80%. The mean of the test was 75 with a standard deviation of 10.

On your English test you score a 75%. The mean is a 70% with a standard deviation of 5.

Which test did you do better on?

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HOMEWORK
get a head start!

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Probabilities in tails: for a normal distribution, use TABLE A, software, or a calculator to find the probability that an observation is

a. At least one standard deviation above the mean

b. At least one standard deviation below the mean

c. In each case, sketch a curve and show the tail probability

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