Bell (Whistle While You) Work!

1. How many ways are there to pick 5 of my 10 best friends to go to Lady Gaga's January concert with me?

$_{10}C_5 = 252$

2. How many ways are there to arrange 10 of my 35 apps on my iPhone? (I care where instagram goes!)

$_{35}P_{10}$

3. How many ways are there to make a playlist of 50 songs from my library of 1,000 songs?

$_{1000}C_{50}$

4. How many ways are there to assign the first 10 parking spaces in the student lot if there are 500 seniors?

$_{500}P_{10}$

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Homework Answers/Questions!

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Get:
- supply bin
- whiteboards & rags (1 per person)

Have out:
- notes
- calc

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5.3 Conditional Probability

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Notes with Whiteboards!

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Multi-Events

More than one event/one way to win

EX/
- P (King followed by Queen)
- Probability of a 1 on the first roll and a 5 on the second roll

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Formulas

P(A then B) P(A) x P(B)

P(A then B then C) P(A) x P(B) x P(C)

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Practice (no replacement)

P(King followed by Q) (4/52) x (4/51) .0060

P(King followed by King) (4/52) x (3/51) .0045

P(King followed by King, then 4) (4/52) x (3/51) x (4/50) .00036

.00036... E-4

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Recall

Independent vs. Dependent

dice
coin
replacement

no replacement

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Recall

Test for Independence

P(A and B) = P(A) x P(B)

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Practice

Are the following independent?

P(A) = 0.4 , P(B) = 0.5 , P(A and B) = 0.6

.4 x .5 = .6
.2 ≠ .6

NO dependent

Is rolling a 2 on the first die and a 3 on the second die independent?

P(2) x P(3) = P(2 & 3)

$\frac{1}{6} \times \frac{1}{6}$

yes

(1/6) x (1/6) = 1/36
1/36 = 1/36
Yuppers!

$\frac{1}{36} = \frac{1}{36}$

2nd: 1 2 3 4 5 6
1st: 1 2 3 4 5 6

$\frac{1}{36}$

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Practice

There are 3 blue marbles and 2 red marbles in a bag

P(red) 2/5 .4

P(blue) 3/5 .6

P(red then blue) replacement
(2/5) x (3/5) .24

Is the above independent?

Yes!

P(R) x P(B) = P(R & B)
.24 = 6/25
.24 = .24 ✓

2nd: B B B R R
1st: B B B R R

$\frac{6}{25}$

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Practice

There are 3 blue marbles and 2 red marbles in a bag

P(red then blue) no replacement (2/5) x (3/4) .3

Is the above independent?

P(R) x P(B) = P(R & B)

Nope! .3 ≠ .24

dependent

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Practice

140 employees were asked about their political affiliation

		Political Affilation		
Employee	Democrat	Republican	Independent	Row Total
Executive (E)	5	34	9	48
Workers (W)	63	21	8	92
Col Totals	68	55	17	140

P(D) 68/140 .49

P(E) 48/140 .34

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Practice

140 employees were asked about their political affiliation

		Political Affilation		
Employee	Democrat	Republican	Independent	Row Total
Executive (E)	5	34	9	48
Workers (W)	63	21	8	92
Col Totals	68	55	17	140

P(D given E)

P(E given D)

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Conditional Probability

Not all data is looked at

P (A given B) is written P(A| B)

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

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Practice

140 employees were asked about their political affiliation

		Political Affilation		
Employee	Democrat	Republican	Independent	Row Total
Executive (E)	5	34	9	48
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Col Totals	68	55	17	140

P(D given E) = 5/48 = .10 5/48

P(E given D) = 5/68 = .074 5/68

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Practice

Eye Color / Hair Color	Brown	Blonde	Black	Red	TOTAL
Brown	75	15	25	1	116
Blue	19	25	0	2	46
Green/Hazel	7	4	2	1	14
TOTAL	101	44	27	4	176

P(Brown Hair|Brown Eyes) 75/116 .65

P(Blue|Blonde) 25/44 .57

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HW #39

Happiness in marriage: Are people happy in their marriages? The table shows results from the 2006 General Social Survey for married adults classified by gender and level of happiness.

	Very Happy	Pretty Happy	Not too Happy	TOTAL
Male	404	221	12	637
Female	457	291	25	773
TOTAL	861	512	37	1410

a. Estimate the probability that a married adult is very happy.

b. Estimate the probability that a married adult is very happy (i) given that their gender is male (ii) given that their gender is female.

c. For these subjects, are the events being very happy and being male independent?

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Bellwork!

GPA	Freshman	Sophomore	Junior	Senior	Total
4.0	14	22	35	30	101
3.0	20	17	18	29	84
2.0	18	25	28	25	96
1.0	20	21	11	7	59
Total	72	85	92	91	340

1. What is the probability that you are a sophomore?
2. What is the probability that your GPA is 4.0?
3. What is the probability that you have a 3.0 given that you are a junior?
4. What is the probability that you are a freshman given that your GPA is a 2.0?

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5.3 Notes continued

Get:

- whiteboards
- supply bin

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Prove for Independence

Two events, A and B, are *independent* if the probability that one occurs is not affected by whether or not the other event occurs.

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Practice

Are events E and D independent?

		Political Affilation		
Employee	Democrat	Republican	Independent	Row Total
Executive (E)	5	34	9	48
Workers (W)	63	21	8	92
Col Totals	68	55	17	140

$P(E|D) = P(E)$

$5/68 \neq 48/140$

No, dependent

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Practice

What was the probability of being audited, given that the income was ≥ $100,000?

TABLE 5.2: Contingency Table Cross-Tabulating Tax Forms by Income Level and Whether Audited

There were 80.2 million forms, and for simplicity the frequencies are reported in thousands and rounded. For example, 90 represents 90,000 tax forms that reported income under $25,000 and were audited.

	Whether Audited		
Income Level	Yes	No	**Total**
Under $25,000	90	14010	**14100**
$25,000–$49,999	71	30629	**30700**
$50,000–$99,999	69	24631	**24700**
$100,000 or more	80	10620	**10700**
Total	**310**	**79890**	**80200**

Source: Statistical Abstract of the United States: 2003.

80/10700 .0075

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Practice

What was the probability of being audited, given that the income is less than $25,000?

TABLE 5.2: Contingency Table Cross-Tabulating Tax Forms by Income Level and Whether Audited

There were 80.2 million forms, and for simplicity the frequencies are reported in thousands and rounded. For example, 90 represents 90,000 tax forms that reported income under $25,000 and were audited.

	Whether Audited		
Income Level	Yes	No	**Total**
Under $25,000	90	14010	**14100**
$25,000–$49,999	71	30629	**30700**
$50,000–$99,999	69	24631	**24700**
$100,000 or more	80	10620	**10700**
Total	**310**	**79890**	**80200**

Source: Statistical Abstract of the United States: 2003.

90/14100 .0064

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Practice

A study of 5282 women aged 35 or over analyzed the Triple Blood Test to test its accuracy.

TABLE 5.5: Contingency Table for Triple Blood Test of Down Syndrome

	Blood Test Result		
Down Syndrome Status	POS	NEG	**Total**
D (Down)	48	6	**54**
D^c (unaffected)	1307	3921	**5228**
Total	**1355**	**3927**	**5282**

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So you know...

A positive test result states that the condition is present.

A negative test result states that the condition is not present.

false positive: test states that the condition is present, but it is actually absent

false negative: test states that the condition is absent but it is actually present

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Practice

TABLE 5.5: Contingency Table for Triple Blood Test of Down Syndrome

	Blood Test Result		
Down Syndrome Status	POS	NEG	**Total**
D (Down)	48	6	**54**
D^c (unaffected)	1307	3921	**5228**
Total	**1355**	**3927**	**5282**

P(Pos) 1355/5282 .26

P(D| Pos) 48/1355 .035

*of the women who tested positive, fewer than 4% actually had fetuses w/ D

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Multiplication Rule

$P(A \text{ and } B) = P(A | B) \times P(B)$

also

$P(A \text{ and } B) = P(B | A) \times P(A)$

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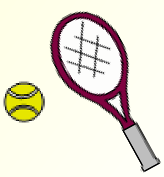
Double Fault

On the first serve, a tennis player has to get the ball to land in a specific region of the court. If it doesn't, he gets a second chance to serve. Usually, the player is much more careful on the second serve and rarely faults again.



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Practice

Roger Federer - 2006 Men's Champion in the Wimbledon tennis tournament

- He made 56% of his first serves
- So, he faulted on the first serve 44% of the time
- Given that he made a fault with his first serve, he made a fault on his second serve only 2% of the time.

What's the probability that he makes a double fault?

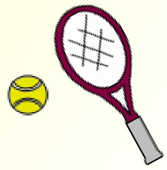
$P(F1) = .44$
$P(F2|F1) = .02$

$P(A \text{ and } B) = P(A|B) \times P(B)$

also

$P(F1 \text{ and } F2) = P(F2|F1) \times P(F1)$

$P(A \text{ and } B) = P(B|A) \times P(A)$



$P(F1 \text{ and } F2) = .44 \times .02$
$= .0088$

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Practice

Sarena Williams

- She made 78% of her first serves
- So, she faulted on the first serve 22% of the time
- Given that she made a fault with her first serve, she made a fault on her second serve only 4% of the time.

What's the probability that she makes a double fault?

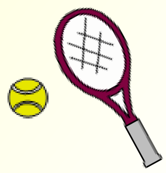
$P(F1) = .22$
$P(F2|F1) = .04$

$P(A \text{ and } B) = P(A|B) \times P(B)$

also

$P(F1 \text{ and } F2) = P(F2|F1) \times P(F1)$

$P(A \text{ and } B) = P(B|A) \times P(A)$



$P(F1 \text{ and } F2) = .22 \times .04$
$= .0088$

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Practice

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Down Syndrome Status	POS	NEG	**Total**
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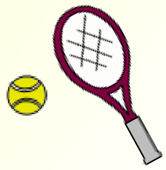
D^c (unaffected)	1307	3921	**5228**
Total	**1355**	**3927**	**5282**

Are POS and D independent?

P(Pos| D) = P(Pos)

48/54 ≠ 1355/5282

No, dependent



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Review
Think/Pair/Share

How do we prove independence?

P(A and B) = P(A) x P(B)

P(A| B) = P(A)

What does Dependent mean?

If P(B) is affected by P(A)

What is the formula for P(A B)

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

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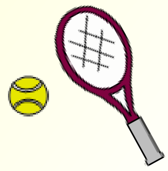
Homework

Pg. 243 # 30, 31, 33, 37, 39, 40, 43, 46



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Nov 14-11:00 AM