Bellwork

In a recent year, 73% of first-year college students responding to a national survey identified "being very well-off financially" as an important personal goal. A state university finds that 132 of an SRS of 200 of its first-year students say that this goal is important. Is there good evidence that the proportion of all first-year students at this university who think being very well-off is important differs from the national value, 73%? Carry out a test at the $\alpha = 0.05$ significance level to help answer this question.

$H_0: p = .73$ $H_a: p \neq .73$ $\alpha = .05$

$x: 132$ $n: 200$

$z = \pm 2.23$ p-value = .0258

$.0258 < .05$ → We Reject H_0 → We conclude that the proportion of students who agree w/ statement is not .73

9.3
Tests About a Population Mean
(Day 1)

vocab

examples

Objectives

3 Conditions to Check

Random: The sample is randomly collected

Independent: The 10% condition must be met

$n \leq .1N$

Normal: $n \geq 30$

* if the sample size is smaller than 30, plot data and check that there is no strong skew or outliers before continuing

Test Statistic

$$\text{test statistic} = \frac{\text{statistic - parameter}}{\text{standard deviation of statistic}}$$

If you know the population standard deviation, use z

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

If you don't know the population standard deviation, use t

$$t = \frac{\bar{x} - \mu_0}{s_x / \sqrt{n}}$$

Better Batteries

The battery company wants to test H_0: $\mu = 30$ versus H_a: $\mu > 30$ based on an SRS of 15 new AAA batteries with mean lifetime $\bar{x} = 33.9$ hours and standard deviation $s_x = 9.8$ hours. Calculate the test statistic.

$$t = \frac{\bar{x} - \mu_0}{S_x / \sqrt{n}} = \frac{(33.9 - 30)}{(9.8 / \sqrt{15})} = \boxed{1.54 = t}$$

Better Batteries

The battery company wants to test H_0: $\mu = 30$ versus H_a: $\mu > 30$ based on an SRS of 15 new AAA batteries with mean lifetime $\bar{x} = 33.9$ hours and standard deviation $s_x = 9.8$ hours. Calculate the test statistic.

$$t = \frac{33.9 - 30}{9.8/\sqrt{15}} = 1.54$$

One Sample T Test

Use the same 4 step process as a One Sample Z Test

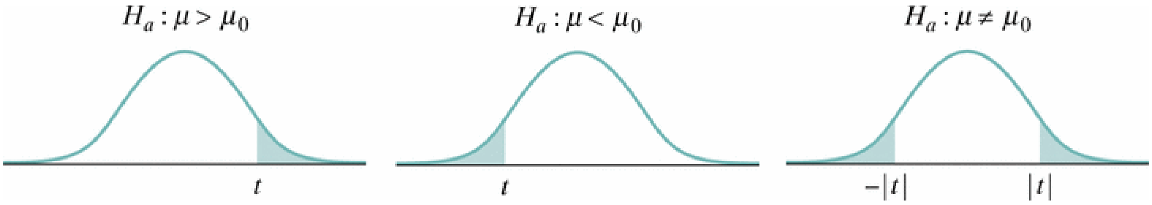
(State, Plan, Do, Conclude)

Choose an SRS of size n from a large population with unknown mean μ. To test the hypothesis H_0: $\mu = \mu_0$, compute the one-sample t statistic:

$$t = \frac{\bar{x} - \mu_0}{s_x / \sqrt{n}}$$

One Sample T Test

Find the P-value by calculating the probability of getting a t statistic this large or larger in the direction specified by the alternative hypothesis H_a in a t distribution with df = n − 1:



Better Batteries

The battery company wants to test H_0: μ = 30 versus H_a: ~~μ > 30~~ based on an SRS of 15 new AAA batteries with mean lifetime $\bar{x}$ = 33.9 hours and standard deviation s_x = 9.8 hours. Calculate the p-value.

t = 1.54 n = 15

df = 15 − 1 = 14

p-value = .0729

Upper-tail probability *p*			
df	**.10**	**.05**	**.025**
13	1.350	1.771	2.160
14	1.345	1.761	2.145
15	1.341	1.753	2.131
	80%	90%	95%
	Confidence level *C*		

calc

table

tcdf(lower, upper, df)

p-value = between .10 & .05

Slow Down Fast Car!

Every road has one at some point—construction zones that have much lower speed limits. To see if drivers obey these lower speed limits, a police officer used a radar gun to measure the speed (in miles per hour, or mph) of a random sample of 10 drivers in a 25 mph construction zone. Here are the results:

27 33 32 21 30 30 29 25 27 34

(a) Is there convincing evidence that the average speed of drivers in this construction zone is greater than the posted 25 mph speed limit?

Slow Down Fast Car!

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27 33 32 21 30 30 29 25 27 34

(a) Is there convincing evidence that the average speed of drivers in this construction zone is greater than the posted 25 mph speed limit?

STATE: H_0: $\mu = 25$ H_a: $\mu > 25$ $\alpha = .05$

μ is the mean speed in mph that the cars are traveling

Slow Down Fast Car!

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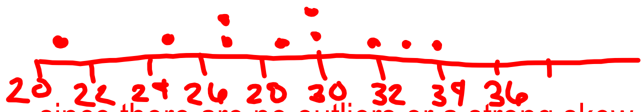
27	33	32	21	30	30	29	25	27	34

(a) Is there convincing evidence that the average speed of drivers in this construction zone is greater than the posted 25 mph speed limit?

PLAN: Random - yes! Independent - assuming at least 100 drivers in the zone

Normal $10 \not> 30$, so plot the data:



since there are no outliers or a strong skew, we will use a one sample t test for mean

Robust

Slow Down Fast Car!

Every road has one at some point—construction zones that have much lower speed limits. To see if drivers obey these lower speed limits, a police officer used a radar gun to measure the speed (in miles per hour, or mph) of a random sample of 10 drivers in a 25 mph construction zone. Here are the results:

27	33	32	21	30	30	29	25	27	34

(a) Is there convincing evidence that the average speed of drivers in this construction zone is greater than the posted 25 mph speed limit?

DO: t = 3.051 p-value = .00689 (show work!)

$$t = \frac{28.8 - 25}{3.9384/\sqrt{10}} = \boxed{3.051 = t}$$

p-value:

tcdf(3.05, 1E99, 9) = $\boxed{.00689}$

lower upper df

Slow Down Fast Car!

Every road has one at some point—construction zones that have much lower speed limits. To see if drivers obey these lower speed limits, a police officer used a radar gun to measure the speed (in miles per hour, or mph) of a random sample of 10 drivers in a 25 mph construction zone. Here are the results:

27	33	32	21	30	30	29	25	27	34

(a) Is there convincing evidence that the average speed of drivers in this construction zone is greater than the posted 25 mph speed limit?

CONCLUDE: .00689 < .05 --> We reject H_0 --> We conclude that the mean speed the cars are driving exceeds 25 mph. The results are statistically significant at the 5% level.

Slow Down Fast Car!

Every road has one at some point—construction zones that have much lower speed limits. To see if drivers obey these lower speed limits, a police officer used a radar gun to measure the speed (in miles per hour, or mph) of a random sample of 10 drivers in a 25 mph construction zone. Here are the results:

27	33	32	21	30	30	29	25	27	34

(a) Is there convincing evidence that the average speed of drivers in this construction zone is greater than the posted 25 mph speed limit?

calculator

Slow Down Fast Car!

Every road has one at some point—construction zones that have much lower speed limits. To see if drivers obey these lower speed limits, a police officer used a radar gun to measure the speed (in miles per hour, or mph) of a random sample of 10 drivers in a 25 mph construction zone. Here are the results:

27 33 32 21 30 30 29 25 27 34

(b) Given your conclusion in part (a), which kind of mistake—a Type I or a Type II error—could you have made? Explain what this mistake means in this context.

9.3
Tests About a Population Mean
(Day 2)

vocab

examples

Slow Down Fast Car!

Every road has one at some point—construction zones that have much lower speed limits. To see if drivers obey these lower speed limits, a police officer used a radar gun to measure the speed (in miles per hour, or mph) of a random sample of 10 drivers in a 25 mph construction zone. Here are the results:

27 33 32 21 30 30 29 25 27 34

(a) Is there convincing evidence that the average speed of drivers in this construction zone is greater than the posted 25 mph speed limit?

calculator

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27 33 32 21 30 30 29 25 27 34

(b) Given your conclusion in part (a), which kind of mistake—a Type I or a Type II error—could you have made? Explain what this mistake means in this context.

Since we concluded about H_a and rejected H_o, it is possible we could have made a Type I error. This would mean that the cars are not actually traveling over 25 mph on average, even though we concluded that they were.

Don't Break the Ice!

In the children's game Don't Break the Ice, small plastic ice cubes are squeezed into a square frame. Each child takes turns tapping out a cube of "ice" with a plastic hammer, hoping that the remaining cubes don't collapse. For the game to work correctly, the cubes must be big enough so that they hold each other in place in the plastic frame but not so big that they are too difficult to tap out. The machine that produces the plastic cubes is designed to make cubes that are 29.5 millimeters (mm) wide, but the actual width varies a little. To ensure that the machine is working well, a supervisor inspects a random sample of 50 cubes every hour and measures their width. The Fathom output summarizes the data from a sample taken during one hour.

(a) Interpret the standard deviation and the standard error provided by the computer output.

Collection 1

Width	29.4943 mm 50 0.0877121 mm 0.0124044 mm

S1 = mean ( )
S2 = count ( )
S3 = stdDev ( )
S4 = stdError ( )

Don't Break the Ice!

(a) Interpret the standard deviation and the standard error provided by the computer output.

The standard deviation, .0877 mm, tells us that the average distance from the mean width of the cubes is .0877 mm for our sample.

The standard error, .0124 mm, tells us that in repeated samples of size 50, our sample mean will be .0124 mm from our true mean.

Collection 1

Width	29.4943 mm
	50
	0.0877121 mm
	0.0124044 mm

S1 = mean ( )
S2 = count ( )
S3 = stdDev ( )
S4 = stdError ( )

Don't Break the Ice!

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(b) Do these data give convincing evidence that the mean width of cubes produced this hour is not 29.5 mm? Use a significance test with $\alpha = 0.05$ to find out.

Collection 1

Width	29.4943 mm
	50
	0.0877121 mm
	0.0124044 mm

S1 = mean ( )
S2 = count ( )
S3 = stdDev ( )
S4 = stdError ( )

Don't Break the Ice!

(b) Do these data give convincing evidence that the mean width of cubes produced this hour is not 29.5 mm? Use a significance test with α = 0.05 to find out.

STATE: H_0: μ = 29.5 H_a: μ ≠ 29.5 α = .05

μ is the mean width of the plastic cubes

PLAN: Random - yes!

Independent - assuming that at least 500 cubes are produced in an hour

Normal - 50 ≥ 30

We will use a one sample t test for mean

Collection 1

Width	29.4943 mm 50 0.0877121 mm 0.0124044 mm

S1 = mean ( )
S2 = count ( )
S3 = stdDev ( )
S4 = stdError ( )

Don't Break the Ice!

(b) Do these data give convincing evidence that the mean width of cubes produced this hour is not 29.5 mm? Use a significance test with α = 0.05 to find out.

DO: on the calculator...

T-test μ_0 : 29.5 $\bar{x}$: 29.4943 s_x: .0877121 n: 50

μ: ≠μ_0

t = -.4595 p-value = .6479

Collection 1

Width	29.4943 mm 50 0.0877121 mm 0.0124044 mm

S1 = mean ( )
S2 = count ( )
S3 = stdDev ( )
S4 = stdError ( )

CONCLUDE: .65 ≥ .05 --> We fail to reject the null hypothesis --> We cannot conclude that the mean width of the plastic cubes is different than 29.5mm. The results are not statistically significant at the 5% level.

Don't Break the Ice!

In the children's game Don't Break the Ice, small plastic ice cubes are squeezed into a square frame. Each child takes turns tapping out a cube of "ice" with a plastic hammer, hoping that the remaining cubes don't collapse. For the game to work correctly, the cubes must be big enough so that they hold each other in place in the plastic frame but not so big that they are too difficult to tap out. The machine that produces the plastic cubes is designed to make cubes that are 29.5 millimeters (mm) wide, but the actual width varies a little. To ensure that the machine is working well, a supervisor inspects a random sample of 50 cubes every hour and measures their width. The Fathom output summarizes the data from a sample taken during one hour.

(c) Calculate a 95% confidence interval for μ. Does your interval support your decision from part (b)?

(29.469, 29.519)

$H_0: \mu = 29.5$

Collection 1

Width	29.4943 mm 50 0.0877121 mm 0.0124044 mm

S1 = mean ( )
S2 = count ( )
S3 = stdDev ( )
S4 = stdError ( )

Don't Break the Ice!

(c) Calculate a 95% confidence interval for μ. Does your interval support your decision from part (b)?

TInterval $\bar{x}$: 29.4943
s_x: .0877121
n: 50
C-level: .95

(29.469, 29.519)

Since the interval captures 29.5mm, this favors our decision in (b)

Collection 1

Width	29.4943 mm 50 0.0877121 mm 0.0124044 mm

S1 = mean ( )
S2 = count ( )
S3 = stdDev ( )
S4 = stdError ( )

Grocery Lines

For their second semester project in AP Statistics, Libby and Kathryn decided to investigate which line was faster in the supermarket: the express lane or the regular lane. To collect their data, they randomly selected 15 times during a week, went to the same store, and bought the same item. However, one of them used the express lane and the other used a regular lane. To decide which lane each of them would use, they flipped a coin. If it was heads, Libby used the express lane and Kathryn used the regular lane. If it was tails, Libby used the regular lane and Kathryn used the express lane. They entered their randomly assigned lanes at the same time, and each recorded the time in seconds it took them to complete the transaction. Carry out a test to see if there is convincing evidence that the express lane is faster.

$H_0: \mu = 0$ $H_a: \mu < 0$

μ is the mean difference (express - regular) in check out time

$\alpha = .05$

Grocery Lines

use difference (express - regular)!

Time in express lane (seconds)	Time in regular lane (seconds)
337	342
226	472
502	456
408	529
151	181
284	339
150	229
357	263
349	332
257	352
321	341
383	397
565	694
363	324
85	127

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STATE: $H_0: \mu = 0$ $H_a: \mu < 0$ $\alpha = .05$

μ is the mean difference in seconds (express lane - regular lane)

Grocery Lines

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PLAN: Independent - assuming at least 150 total possible trips to the grocery store Random - yes! Normal - $15 \ngtr 30$ so we plot the data (show dot plot)

Since there are no outliers or extreme skew, we will use a one sample t test for means

Grocery Lines

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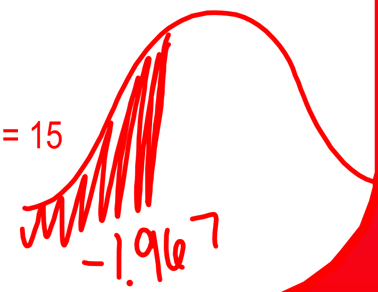
DO: on the calculator...

μ_0: 0 μ: < 0 $\bar{x}$ = -42.67 s_x = 84.019 n = 15

t = -1.967 p-value = .0346



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CONCLUDE: .0346 < .05 --> We reject H_0 --> We conclude that the average time is less than 0 seconds and so the express lane is faster. The results are statistically significant at the 5% level.

Homework

9.3 assignment