Bellwork

A contract between a manufacturer and a consumer of light bulbs specifies that
with an SRS of 100 bulbs.

(a) Describe what a Type I error would be in this context.

(b) Describe what a Type II error would be in this context.

Bellwork

A contract between a manufacturer and a consumer of light bulbs specifies that the mean lifetime of the bulbs must be at least 1000 hours. As part of the quality assurance program, the manufacturer will institute an inspection program for each day's production of 10,000 units. An ordinary testing procedure is difficult since 1000 hours is over 41 days! Since the lifetime of a bulb decreases as the voltage applied increases, a common procedure is to perform an accelerated lifetime test in which the bulbs are lit using 400 volts (compared to the usual 110 volts). At such a voltage, a 1000-hour bulb is expected to last only 3 hours. This is a well-known procedure, and both sides have agreed that the results from the accelerated test will be a valid indicator of lifetime of the bulb. The manufacturer will test the hypotheses $H_0 : \mu = 3$ *versus* $H_a : \mu < 3$ at the a = 0.01 level
with an SRS of 100 bulbs.

(a) Describe what a Type I error would be in this context.

(b) Describe what a Type II error would be in this context.

$\alpha = .01$ $\alpha = .05$

Pvalue = .002

.031

$\alpha = .05$

.01

9.2
Tests About a
Population Proportion
(Day 1)

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Objectives

Conditions for a Test

Random: Subjects are selected at random

Normal: $n(p) \geq 10$

$n(1-p) \geq 10$

*note, we always test assuming H_O is true so use p instead of $\hat{p}$ (p_o)

Independent: The 10% condition is met

$n \leq .1\ N$

Conditions for a Test

When the three conditions are met, the distribution is approximately Normal with:

mean: $\mu_{\hat{p}} = p$ (p_0)

standard deviation: $\sigma_{\hat{p}} = \sqrt{\dfrac{p(1-p)}{n}}$ (p_0)

B-Baller

Remember our virtual B-Ball Player?

We took an SRS of 50 of his shots and found that he only made 32.

His claim was that he made 80% of his shots:

H_O: $p = .8$

H_a: $p \,\cancel{>}\, < .8$

Are the conditions met?

Random: yes SRS

Ind: Assuming at least 500 shots total

Normal: $50(.8) \geq 10$
$50(1-.8) \geq 10$ ✓ yes

Test Statistic

measures how far a sample statistic diverges from what we would expect if the null hypothesis were true, in standardized units

$$z = \text{test statistic} = \frac{\overset{x - \mu_{\hat{p}}}{\text{statistic - parameter}}}{\underset{\sigma_{\hat{p}}}{\text{standard deviation of statistic}}}$$

On Formula Sheet! (Back Side)

Test Statistic

What do we use for the standard deviation of the statistic?

$\mu_{\hat{p}}$ $\sigma_{\hat{p}}$

4 Step Process!

State: H_o & H_a α =

Define parameters!

Plan: Choose inference method (one sample z test for proportion)

Check 3 conditions

Do: Compute test statistic

Find P-Value

Conclude: Interpret the results in context

One Sample z Test for a Proportion

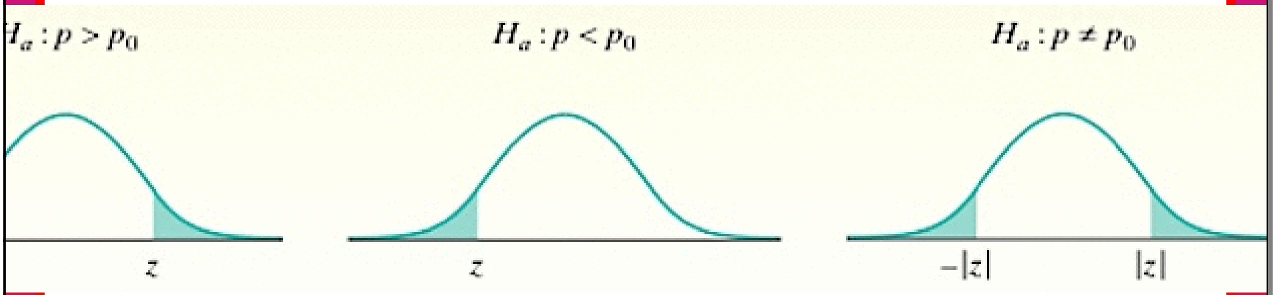
Choose an SRS of size n from a large population that contains an unknown proportion p of successes. To test the hypothesis H_0: p = $\hat{p}$, compute the z statistic:

$$z = \frac{\hat{p} - p_o}{\sqrt{\frac{p_o(1-p_o)}{n}}}$$

$$\text{test statistic} = \frac{\text{statistic} - \text{parameter}}{\text{st. dev.}}$$

One Sample z Test for a Proportion

Find the P-value by calculating the probability of getting a z statistic this large or larger in the direction specified by the alternative hypothesis H_a:



normalcdf p-value

One Potato, Two Potato

The potato-chip producer of Section 9.1 has just received a truckload of potatoes from its main supplier. Recall that if the producer determines that more than 8% of the potatoes in the shipment have blemishes, the truck will be sent away to get another load from the supplier. A supervisor selects a random sample of 500 potatoes from the truck. An inspection reveals that 47 of the potatoes have blemishes. Carry out a significance test at the α = 0.10 significance level. What should the producer conclude?

STATE:

$H_0: p = .08$ $\alpha = .1$

$H_a: p > .08$

p is proportion of potatoes that are blemished

One Potato, Two Potato

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PLAN:

Use one sample z test for proportion

Random: yes!

Normal: $500(.08) \geq 10$ ✓

$500(1-.08) \geq 10$

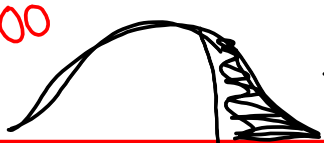
Ind: assuming at least 5000 potatoes

One Potato, Two Potato

The potato-chip producer of Section 9.1 has just received a truckload of potatoes from its main supplier. Recall that if the producer determines that more than 8% of the potatoes in the shipment have blemishes, the truck will be sent away to get another load from the supplier. A supervisor selects a random sample of 500 potatoes from the truck. An inspection reveals that 47 of the potatoes have blemishes. Carry out a significance test at the α = 0.10 significance level. What should the producer conclude?

DO:

$$z = \frac{.094 - .08}{\sqrt{\frac{.08(1-.08)}{500}}} = \boxed{1.15 = z}$$

P-value:  $= \boxed{.125}$

$$\text{normalcdf}(\underset{\text{min}}{\overset{1.15}{1.15}}, \underset{\text{max}}{1E99}, \underset{\mu}{0}, \underset{\sigma}{1})$$

One Potato, Two Potato

The potato-chip producer of Section 9.1 has just received a truckload of potatoes from its main supplier. Recall that if the producer determines that more than 8% of the potatoes in the shipment have blemishes, the truck will be sent away to get another load from the supplier. A supervisor selects a random sample of 500 potatoes from the truck. An inspection reveals that 47 of the potatoes have blemishes. Carry out a significance test at the α = 0.10 significance level. What should the producer conclude?

CONCLUDE:

$.125 \geq .10 \rightarrow$ We fail to Reject H_0

$\rightarrow$ We can't conclude that the proportion of blemished potatoes is greater than 8%. The Results are not statistically significant at the 10% level.

On the calculator...

Better to be last?

On shows like American Idol, contestants often wonder if there is an advantage to performing last. To investigate this, a random sample of 600 American Idol fans is selected, and they are shown the audition tapes of 12 never-before-seen contestants. For each fan, the order of the 12 videos is randomly determined. Thus, if the order of performance doesn't matter, we would expect approximately 1/12 of the fans to prefer the last contestant they view. In this study, 59 of the 600 fans preferred the last contestant they viewed. Do these data provide convincing evidence that there is an advantage to going last at a significance level of .05?

9.2
Tests About a Population Proportion
(Day 2)

vocab

examples

Better to be last?

On shows like American Idol, contestants often wonder if there is an advantage to performing last. To investigate this, a random sample of 600 American Idol fans is selected, and they are shown the audition tapes of 12 never-before-seen contestants. For each fan, the order of the 12 videos is randomly determined. Thus, if the order of performance doesn't matter, we would expect approximately 1/12 of the fans to prefer the last contestant they view. In this study, 59 of the 600 fans preferred the last contestant they viewed. Do these data provide convincing evidence that there is an advantage to going last at a significance level of .05?

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STATE: H_o: $p = 1/12$ (.083) H_a: $p > 1/12$ (.083) $\alpha = .05$

p is the proportion of American Idol fans who prefer the last performer

Better to be last?

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PLAN: Random: yes

Independent: assuming more than 6000 viewers

Normal: 600(1/12) ≥ 10

600 (11/12) ≥ 10 yes!

We will use a one sample z test for proportion

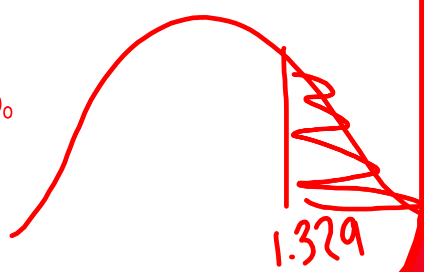
Better to be last?

On shows like American Idol, contestants often wonder if there is an advantage to performing last. To investigate this, a random sample of 600 American Idol fans is selected, and they are shown the audition tapes of 12 never-before-seen contestants. For each fan, the order of the 12 videos is randomly determined. Thus, if the order of performance doesn't matter, we would expect approximately 1/12 of the fans to prefer the last contestant they view. In this study, 59 of the 600 fans preferred the last contestant they viewed. Do these data provide convincing evidence that there is an advantage to going last at a significance level of .05?

DO: On the calculator...

1-PropZTest p_o: 1/12 x: 59 n: 600 $>p_o$

z = 1.329 p-value = .0918 $\hat{p}$ = .0983



Better to be last?

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CONCLUDE: .0918 ≥ .05 --> We fail to reject H_o --> We can't conclude that fans prefer the last performance more often. The results are not statistically significant at the 5% level.

Benford's Law

When the accounting firm AJL and Associates audits a company's financial records for fraud, they often use a test based on Benford's law. Benford's law states that the distribution of first digits in many real-life sources of data is not uniform. In fact, when there is no fraud, about 30.1% of the numbers in financial records begin with the digit 1. However, if the proportion of first digits that are 1 is significantly different from 0.301 in a random sample of records, AJL and Associates does a much more thorough investigation of the company. Suppose that a random sample of 300 expenses from a company's financial records results in only 68 expenses that begin with the digit 1. Should AJL and Associates do a more thorough investigation of this company? Use an significance level of .05.

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STATE: H_o: $p = .301$ H_a: $p \neq .301$ $\alpha = .05$

p is the proportion of first digits that are 1

Benford's Law

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PLAN: Random - yes! Independent - assuming at least 3000 expenses

Normal - $300(.301) \geq 10$ & $300(1-.301) \geq 10$ yes!

We will use a one sample z test for proportion

Benford's Law

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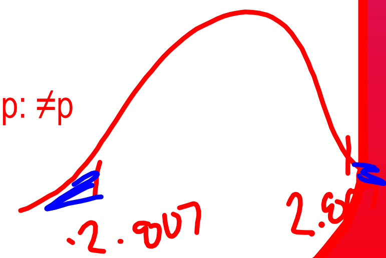
DO: On the calculator...

1-PropZTest p_0: .301 x: 68 n: 300 prop: ≠p

z = -2.807 p-value = .005 $\hat{p}$ = .2267



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CONCLUDE: .005 < .05 --> We reject H_0 --> We conclude that the proportion of first digits that are 1 does not equal .301. The results are statistically significant at the 5% levle. AJL should do a more thorough investigation.

Benford's Law

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Describe a Type I and Type II error in this context.

Benford's Law

Describe a Type I and Type II error in this context.

A Type I Error would be concluding that the proportion of first digits that are 1 is not equal to .301 when in fact it is.

A Type II Error would be NOT concluding that the proportion of first digits that are 1 is not equal to .301 when in fact it actually is different from .301.

Using Confidence Intervals

There is a link between confidence intervals and the decision to reject or fail to reject H_o

For example, a 95% confidence interval would give an approximate range of p_o's that would not be rejected for a two-sided test at an $\alpha = 0.05$ level

Confidence Intervals are not a perfect representation, but close, since they use values of $\hat{p}$ not p_o

Benford's Law

A 95% confidence interval for the true proportion of expenses that begin with the digit 1 for the company in the previous Example is (0.180, 0.274). Does the interval provide convincing evidence that the company should be investigated for fraud?

$H_0: p = .301$ Reject

✓ $H_a: \boxed{p \neq .301}$

Computer Output

Session

Test and Confidence Interval for One Proportion

Test of p = 0.75 vs p not = 0.75

~~Sample~~	X	N	Sample p	95.0 % CI	Z-Value	P-Value
~~1~~	68	100	0.680000	(0.588572, 0.771428)	-1.62	0.106
			$\hat{p}$			

Homework

9.2 Assignment (due Thursday)