

Bellwork

Crop researchers are interested in the productivity of a new variety of corn. They plant 123 plots with randomly-selected seeds of the new variety, record the yield in bushels per acre, and find that a 99% confidence interval for the true mean yield is 118 to 130 bushels per acre.

- (a) What is the point estimate from this sample? 124
- (b) What is the margin of error? ± 6
- (c) Interpret the 99% confidence *interval* 118 to 130 in the context of the problem.
- (d) Interpret the confidence *level* of 99% in the context of the problem.

99% of all possible samples of size 123 will result in an interval that captures the true pop. mean yield of corn.

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8.2 Estimating a Population Proportion (Day 1)

vocab

examples

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Objectives

- Construct and interpret a confidence interval for a population proportion
- Determine critical values for calculating a confidence interval using a table or calculator
- Carry out the steps in constructing a confidence interval for a population proportion: define the parameter, check conditions, perform calculations, interpret results in context
- Determine the sample size required to obtain a level C confidence interval for a population proportion with a specified margin of error
- Understand how the margin of error of a confidence interval changes with the sample size and the level of confidence C

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Beads!

In my bag there are A LOT of beads! Our goal is to estimate the actual proportion of purple beads in the bag (since there are too many to count).

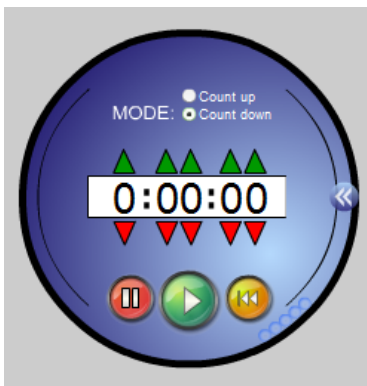
1. How can we use the cup provided to get a SRS of beads from the container? Think this through carefully, because we will only take 1 sample.
2. Let's take the sample. Count the # purple and the total #
3. Determine a point estimate for the unknown population proportion p of purple beads in the container.

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Beads!

4. Challenge! Each group work on a method to construct a 90% confidence interval for the parameter p .

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = .025$$



$$.0885 - .1525$$

5. Compare results and discuss any problems/differences.

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Conditions for Constructing a Confidence Interval

Normal: The sample size must be large enough that both np and $n(1-p)$ are at least 10

*note: we use \hat{p} instead of p (since we don't actually know p)

Random: The sample has been randomly collected

Independent: The 10% condition must be met

$$n \leq .1N$$

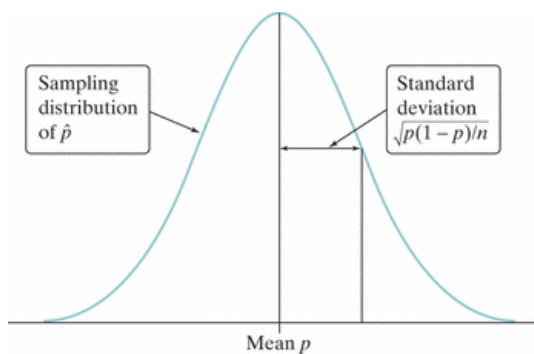
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Conditions for Constructing a Confidence Interval

If the conditions are met, then the Normal curve is defined by:

$$\text{Mean} = p \quad \mu_{\hat{p}}$$

$$\text{Standard deviation} = \frac{\sqrt{p(1-p)}}{n} \quad \sigma_{\hat{p}}$$



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Beads

Check that the conditions for constructing a confidence interval for p are met for our bead activity!

Normal:

Random:

Independent:

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Beads

What is the standard deviation of the sample proportion of red beads?

*Since we don't know p , what can we replace it with?

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Standard Error

When the standard deviation of a statistic is estimated from the data

(i.e. When we replace p with \hat{p} to calculate the standard deviation of the sample proportion)

notation: $SE_{\hat{p}}$

formula sheet?

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

$$SE_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

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Critical Value

The number of standard deviations you want to be on either side of the mean corresponding to your confidence level

notation: z^*

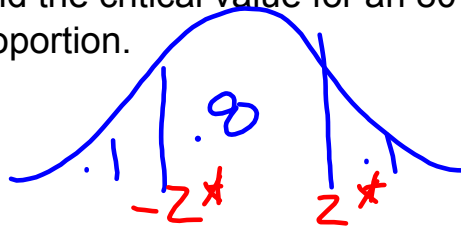
(* reminds us that it's not calculated from actual data like other z-scores)

Find the critical value using Table A or invNorm on the calculator

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Critical Value

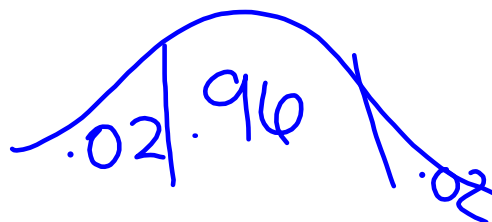
Find the critical value for an 80% confidence interval for a proportion.



$$-z^* = -1.28$$

$$z^* = 1.28$$

Find the critical value for a 96% confidence interval for a proportion.



$$-z^* = -2.05$$

$$z^* = 2.05$$

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Critical Value

Find the critical value for an 80% confidence interval for a proportion.

Find the critical value for a 96% confidence interval for a proportion.

$$\text{invNorm}(.02,0,1) = -2.05 = -z^*$$

$$2.05 = z^*$$

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8.2 Estimating a Population Proportion (Day 2)

vocab

examples

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Objectives

- Construct and interpret a confidence interval for a population proportion
- Determine critical values for calculating a confidence interval using a table or calculator
- Carry out the steps in constructing a confidence interval for a population proportion: define the parameter, check conditions, perform calculations, interpret results in context
- Determine the sample size required to obtain a level C confidence interval for a population proportion with a specified margin of error
- Understand how the margin of error of a confidence interval changes with the sample size and the level of confidence C

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One-Sample z Interval for a Population Proportion

Choose an SRS of size n from a large population that contains an unknown proportion p of successes. An approximate level C confidence interval for p is: *C.V.*

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

stat → \hat{p} z^* $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ ← *SE / st. dev.*

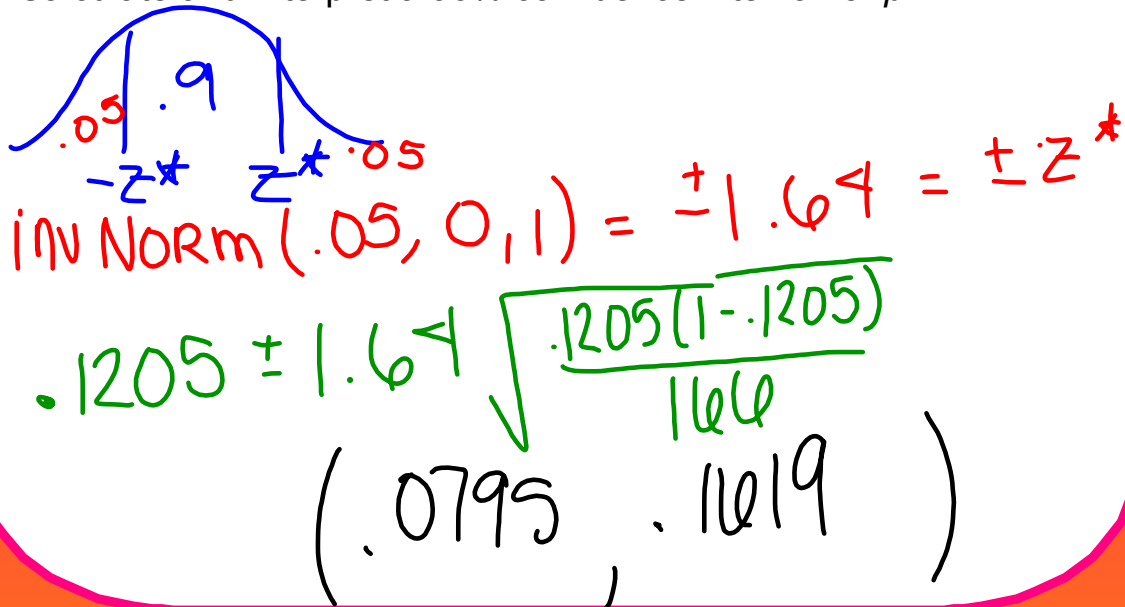
where z^* is the critical value for the standard Normal curve with area C between $-z^*$ and z^* .

note: Use this only when the number of successes and failures in the sample are both at least 10 and the 10% condition is satisfied

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Back to the Beads...

Calculate and Interpret a 90% confidence interval for p .



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Penny for your thoughts...

Students in an AP Statistics class want to estimate the proportion of pennies in circulation that are more than 10 years old. To do this, they took an SRS of a set of pennies. Overall, 57 of the 102 pennies they have are more than 10 years old.

- Identify the population and the parameter of interest.
- Check the conditions for calculating a confidence interval for the parameter.

$$\text{Normal: } n\hat{p} > 10 \text{ \& } n(1-\hat{p}) > 10$$
- Construct a 99% confidence interval for the parameter

$$(.43219, .65546)$$
- Interpret the interval in context.
- Is it possible that more than 60% of all pennies in circulation are more than 10 years old?

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(a) Identify the population and the parameter of interest.

population - all pennies in circulation

parameter - population proportion, p

(b) Check the conditions for calculating a confidence interval for the parameter.

Normal: $np \geq 10 \rightarrow 102(57/102) = 57 \geq 10$

$n(1-p) \geq 10 \rightarrow 102(45/102) = 45 \geq 10$ YES!

Random: Yes, SRS

Independent: $n \leq .1N \rightarrow 102 \leq .1N \rightarrow 1020 \leq N$

Yes, there are more than 1020 pennies in circulation

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(c) Construct a 99% confidence interval for the parameter.

$\text{invNorm}(.005, 0, 1) = -2.58 = -z^*$ $z^* = 2.58$

1 prop z int $X: 57$
 $n: 102$
 $C\text{-Level}: .99$

$(.4322, .6857)$

(d) Interpret the interval in context.

We are 99% confident that the interval from .4322 to .6857 contains the true population proportion of pennies in circulation that are more than 10 years old.

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(e) Is it possible that more than 60% of all pennies in circulation are more than 10 years old?

Yes, since we used a 99% confidence level, we are very confident and our interval goes up to 68.57%. So, it is possible that more than 60% of all pennies in circulation are more than 10 years old.

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Construct and Interpret a Confidence Interval

4 steps!

- State:** What *parameter* do you want to estimate?
At what confidence level?
- Plan:** Are the conditions met?
What is the inference method that you are using?
- Do:** Calculation with full labels
- Conclude:** Interpret your interval in context

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One-Sample z Interval for a Population Proportion

```
1-PropZInt  
x:246  
n:439  
C-Level:.95  
Calculate
```

note: be careful to fully label and still understand the formula as a MC question may contain parts of the formula

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AP TIP!

You may use your calculator to compute a confidence interval, but there's risk involved!

You must show all steps and fully label!

You must name the procedure and state the interval!

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Kissing the right way?

According to an article in the *San Gabriel Valley Tribune* (February 13, 2003), "Most people are kissing the 'right way.'" That is, according to the study, the majority of couples tilt their heads to the right when kissing. In the study, a researcher observed a random sample 124 couples kissing in various public places and found that 83/124 (66.9%) of the couples tilted to the right. Construct and interpret a 95% confidence interval for the proportion of all couples who tilt their heads to the right when kissing.

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STATE: We want to est. the proportion of all couples who turn their head to the right when kissing at a 95% confidence level

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PLAN: We can use a one sample Z interval test for pop. proportion

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8.2 Estimating a Population Proportion (Day 3)

vocab

examples

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Kissing the right way?

According to an article in the *San Gabriel Valley Tribune* (February 13, 2003), "Most people are kissing the 'right way.'" That is, according to the study, the majority of couples tilt their heads to the right when kissing. In the study, a researcher observed a random sample 124 couples kissing in various public places and found that 83/124 (66.9%) of the couples tilted to the right. Construct and interpret a 95% confidence interval for the proportion of all couples who tilt their heads to the right when kissing.

CONCLUDE: We are 95% confident that the interval from .5869 to .7521 will contain the true proportion of couples who kiss to the right.

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Sample Size for Desired Margin of Error

To determine the sample size n that will yield a level C confidence interval for a population proportion p with a maximum margin of error ME , solve the following inequality for n :

$$z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq ME$$

where \hat{p} is a guessed value for the sample proportion. The margin of error will always be less than or equal to ME if you take the guess \hat{p} to be 0.5.

Not on formula sheet, however individual pieces are...

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Tattoos

Suppose that you wanted to estimate p = the true proportion of students at your school who have a tattoo with 98% confidence and a margin of error of no more than 0.10. How many students should you survey?

$$z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq ME$$

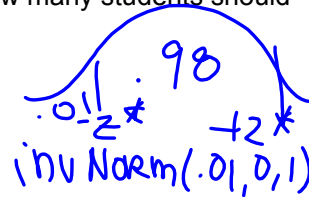
$$\frac{2.33}{2.33} \sqrt{\frac{.5(1-.5)}{n}} \leq \frac{.1}{2.33}$$

$$\left(\frac{.5(1-.5)}{n} \right)^2 \leq (.0433)^2$$

$$\frac{.5(1-.5)}{n} \leq \frac{.0019}{1}$$

$$.25 \leq .0019n$$

$$\boxed{138.88 \leq n}$$



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Tattoos

Suppose that you wanted to estimate p = the true proportion of students at your school who have a tattoo with 98% confidence and a margin of error of no more than 0.10. How many students should you survey?

$$\text{invNorm}(.01, 0, 1) = -2.33 = -z^* \\ 2.33 = z^*$$

$$\text{guess } \hat{p} = .5$$

$$135.87 \leq n \quad \text{We should survey at least 136 students}$$

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One last practice...

A New York Times poll on women's issues interviewed 1025 women randomly selected from the United States, excluding Alaska and Hawaii. The poll found that 482 of the women said they do not get enough time for themselves. Construct and interpret a 90% confidence interval that estimates the proportion of women in the United States who do not feel that they get enough time for themselves. Use the Four-step process.

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STATE: We want to estimate the population proportion of US women, excluding Alaska and Hawaii, who say they do not get enough time for themselves at a 90% confidence level.

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One last practice...

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*PLAN: Normal --> $1025(.47) \geq 10$ & $1025(1-.47) \geq 10$ --> Yes, Normal
Independent --> $1025 < .1N$ --> $10250 < N$ --> Yes, Independent:
we assume there are more than 10,250 women*

Random --> Yes, random

We will use a 1 sample z interval test for a population proportion

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One last practice...

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DO: On the calculator:

1-PropZInt x: 482 n: 1025 C-Level: .9

(.4446, .49589)

CONCLUDE: We are 90% confident that the interval from .4446 to .49589 contains the true population proportion of US women, excluding Alaska and Hawaii, who feel that they don't get enough time for themselves.

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Quiz!

In January 2010, a Gallup Poll asked a random sample of adults, "In general, are you satisfied with the way things are going in the United States at this time?" In all, 256 said that they were satisfied out of 1025 people asked. Construct and interpret a 90% confidence interval for the proportion of adults who are satisfied with how things are going. Follow the four-step process!

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