6.1
Discrete & Continuous
Random Variables

Objectives

Today we will...

- Compute probabilities using the probability distribution of a discrete random variable.

- Calculate and interpret the mean (expected value) of a discrete random variable.
Random Variable

takes numerical values that describe the outcomes of some chance process

ex/ how many tails you get after you flip a coin 3 times

Probability Distribution

lists possible values and their probabilities

<table>
<thead>
<tr>
<th># of heads</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>probability</td>
<td>1/8</td>
<td>3/8</td>
<td>3/8</td>
<td>1/8</td>
</tr>
</tbody>
</table>
Discrete Random Variable

X

takes a fixed set of possible values with gaps between

ex/ number of siblings, number of 6's rolled

ex/ shoe size vs. length of foot

the probability distribution of a discrete random variable lists the values $x_i$ and their probabilities $p_i$

<table>
<thead>
<tr>
<th>$X$</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(X)$</td>
<td>$p_1$</td>
<td>$p_2$</td>
<td>$p_3$</td>
<td>...</td>
</tr>
</tbody>
</table>

the probabilities must satisfy two requirements:
1) Every $p_i$ is between 0 and 1
2) The sum of the probabilities is 1
Discrete Random Variable

to find the probability of any event, add the probabilities of the particular values of \( x_i \)

NHL Goals

In 2010, there were 1319 games played in the National Hockey League’s regular season. Imagine selecting one of these games at random and then randomly selecting one of the two teams that played in the game. Define the random variable \( X \) = number of goals scored by a randomly selected team in a randomly selected game. The table below gives the probability distribution of \( X \):

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P(x) )</td>
<td>0.061</td>
<td>0.154</td>
<td>0.228</td>
<td>0.229</td>
<td>0.173</td>
<td>0.094</td>
<td>0.041</td>
<td>0.015</td>
<td>0.004</td>
<td>0.001</td>
</tr>
</tbody>
</table>

(a) Show that the probability distribution for \( X \) is legitimate.
(b) Make a histogram of the probability distribution. Describe what you see.
(c) What is the probability that the number of goals scored by a team in a randomly selected game is at least 6? More than 6?

\[
P(X > 6) = 0.051 + 0.154 + 0.094 + 0.015 = 0.32\]

\[
P(X > 6) = 0.154 + 0.094 + 0.015 = 0.26\]
NHL Goals

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(x)</td>
<td>0.061</td>
<td>0.154</td>
<td>0.228</td>
<td>0.229</td>
<td>0.173</td>
<td>0.094</td>
<td>0.041</td>
<td>0.015</td>
<td>0.004</td>
<td>0.001</td>
</tr>
</tbody>
</table>

(a) Show that the probability distribution for X is legitimate.

\[ 0 \leq P(x) \leq 1 \quad \text{all probabilities are between 0 and 1} \]

\[ 0.061 + 0.154 + 0.228 + 0.229 + 0.173 + 0.094 + 0.041 + 0.015 + 0.004 + 0.001 = 1 \]

(b) Make a histogram of the probability distribution. Describe what you see.

(see board)

S - The distribution is skewed to the right. O - There are not any outliers. C - The center may be around 3 or 4. S - The probability distribution spans from 0 to 9 goals.

(c) What is the probability that the number of goals scored by a team in a randomly selected game is at least 6? More than 6?

\[ P(x \geq 6) = 0.041 + 0.015 + 0.004 + 0.001 = 0.061 \]

\[ P(x > 6) = 0.015 + 0.004 + 0.001 = 0.02 \]

---

Roulette

One wager players can make in Roulette is called a “corner bet.” To make this bet, a player places his chips on the intersection of four numbered squares on the Roulette table. If one of these numbers comes up on the wheel and the player bet $1, the player gets his $1 back plus $8 more. Otherwise, the casino keeps the original $1 bet. If \( X \) = net gain from a single $1 corner bet, the possible outcomes are \( x = -1 \) or \( x = 8 \). Here is the probability distribution of \( X \):

<table>
<thead>
<tr>
<th>Value</th>
<th>$1</th>
<th>$8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>( \frac{34}{38} )</td>
<td>( \frac{1}{38} )</td>
</tr>
</tbody>
</table>

If a player were to make this $1 bet over and over, what would be the player's average gain?

\[ \mu_x = -1 \left( \frac{34}{38} \right) + 8 \left( \frac{1}{38} \right) = \$ -0.05 \]
Mean (Expected Value)

for discrete random variables!

suppose that \( x \) is a discrete random variable whose probability distribution is:

\[
\begin{array}{c|c|c|c|c}
\text{value} & x_1 & x_2 & x_3 & \ldots \\
\text{prob.} & p_1 & p_2 & p_3 & \ldots \\
\end{array}
\]

the mean (expected value) is found by multiplying each possible value by its probability and adding all the products.

\[
\mu_x = E(x) = x_1p_1 + x_2p_2 + x_3p_3 + \ldots = \sum x_ip_i
\]

*on formula sheet*
Interpreting Mean (Expected Value)

the long-run average after many, many repetitions

\[ \mu_x = -1 \cdot \frac{34}{38} + 8 \cdot \frac{4}{38} \]

\[ = -0.05 \]

The long run winnings (or losses in this case), after many, many games is -$0.05
NHL Goals

Find and interpret the mean of the NHL goals.

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
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<td>0.061</td>
<td>0.154</td>
<td>0.228</td>
<td>0.229</td>
<td>0.173</td>
<td>0.094</td>
<td>0.041</td>
<td>0.015</td>
<td>0.004</td>
<td>0.001</td>
</tr>
</tbody>
</table>

\[
\mu_x = (0 \times 0.061) + (1 \times 0.154) + (2 \times 0.228) + (3 \times 0.229) + \\
(4 \times 0.173) + (5 \times 0.094) + (6 \times 0.041) + \\
(7 \times 0.015) + (8 \times 0.004) + (9 \times 0.001)
\]

\[
\mu_x = 2.851 \text{ goals}
\]

The long-run average amount of goals, over many, many games is 2.851.
AP TIP!

Does the mean have to equal one of the possible values?

No!

Make sure to round out to the decimals

Just leaving the expected value as an integer (if not an integer) will result in an incorrect answer

Ticket Out!

In 1952, Dr. Virginia Apgar suggested five criteria for measuring a baby’s health at birth: skin color, heart rate, muscle tone, breathing, and response when stimulated. She developed a 0-1-2 scale to rate a newborn on each of the five criteria. A baby’s Apgar score is the sum of the ratings on each of the five scales, which gives a whole-number from 0 to 10. Apgar scores are still used today to evaluate the health of newborns. Here is the probability distribution of Apgar scores:

<table>
<thead>
<tr>
<th>Value</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.001</td>
</tr>
<tr>
<td>1</td>
<td>0.006</td>
</tr>
<tr>
<td>2</td>
<td>0.007</td>
</tr>
<tr>
<td>3</td>
<td>0.008</td>
</tr>
<tr>
<td>4</td>
<td>0.012</td>
</tr>
<tr>
<td>5</td>
<td>0.020</td>
</tr>
<tr>
<td>6</td>
<td>0.038</td>
</tr>
<tr>
<td>7</td>
<td>0.099</td>
</tr>
<tr>
<td>8</td>
<td>0.319</td>
</tr>
<tr>
<td>9</td>
<td>0.437</td>
</tr>
<tr>
<td>10</td>
<td>0.053</td>
</tr>
</tbody>
</table>

Find and interpret the expected Apgar value.
Homework

6.1 (day 1) assignment

Bellwork

**Roulette Red/Black Bet** - Suppose that a player places a simple $1 bet on red. If the ball lands in a red slot, the player gets the original dollar back, plus an additional dollar for winning the bet. If the ball lands in a different-colored slot, the player loses the dollar bet to the casino. Here is the probability distribution:

<table>
<thead>
<tr>
<th>Value</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-1$</td>
<td>20/38</td>
</tr>
<tr>
<td>$1$</td>
<td>18/38</td>
</tr>
</tbody>
</table>

What is the expected value of the winnings?

\[ E[X] = (-1) \times \frac{20}{38} + 1 \times \frac{18}{38} \]

\[ E[X] = -0.05 = -\$1 \times \frac{20}{38} + 1 \times \frac{18}{38} \]

How does this compare to the corner bet from yesterday's notes?

\[ -0.05 \]
6) \[ M_x = (1 \cdot 0.301) + 2(1.176) + 3(1.125) + 4(0.911) + 5(0.079) + 6(0.067) + 7(0.058) + 8(0.051) + 9(0.046) \]
\[ M_x = 3.411 \]

The long run avg. after many many words typed is 2.1 ERRORS
6.1
Discrete & Continuous
Random Variables
(Day 2)

Objectives

Today we will...

- calculate the standard deviation and variance of a probability distribution
Variance

suppose that $x$ is a **discrete random variable** whose probability distribution is:

<table>
<thead>
<tr>
<th>value</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>prob.</td>
<td>$p_1$</td>
<td>$p_2$</td>
<td>$p_3$</td>
<td>...</td>
</tr>
</tbody>
</table>

and that $\mu_x$ is the mean of $X$

The variance of $X$ is:

$$Var(X) = \sigma^2 = \sum (x_i - \mu_x)^2 p_i$$

**Standard Deviation**

The standard deviation of $X$, $\sigma_x$, is the square root of the variance.

$$\sigma_x = \sqrt{\Sigma (x_i - \mu_x)^2 p_i}$$
**Interpreting Standard Deviation**

The average distance the outcomes are from the mean.

---

**Roulette**

The "red/black" and "corner" bets in Roulette both had the same expected value. How do you think their standard deviations compare? Calculate them both to confirm your answer.

<table>
<thead>
<tr>
<th>Red/Black</th>
<th>Corner</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value:</td>
<td>Value:</td>
</tr>
<tr>
<td>$-1$</td>
<td>$-1$</td>
</tr>
<tr>
<td>$1$</td>
<td>$8$</td>
</tr>
<tr>
<td>Probability:</td>
<td>Probability:</td>
</tr>
<tr>
<td>$20/38$</td>
<td>$34/38$</td>
</tr>
<tr>
<td>$18/38$</td>
<td>$4/38$</td>
</tr>
</tbody>
</table>

\[
\sigma_x = \sqrt{(-1 + 0.05)^2 (20/38) + (1 + 0.05)^2 (18/38)} = 2.76
\]

\[
\sigma_x = \sqrt{1.00} = 1.00
\]
Roulette

Red/Black

<table>
<thead>
<tr>
<th>Value</th>
<th>-$1</th>
<th>$1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>20/38</td>
<td>18/38</td>
</tr>
</tbody>
</table>

\[ \sigma_x = \sqrt{(-1+.05)^2(20/38) + (1+.05)^2(18/38)} = $1.00 \]

Corner

<table>
<thead>
<tr>
<th>Value:</th>
<th>-$1</th>
<th>$8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability:</td>
<td>34/38</td>
<td>4/38</td>
</tr>
</tbody>
</table>

\[ \sigma_x = \sqrt{(-1+.05)^2(34/38) + (1+.05)^2(4/38)} = $2.76 \]

Can we use our calculator for this?

Somewhat...
AP TIP!

When you are expected to calculate the mean or standard deviation of a random variable, you must show adequate work.

You can't just report calculator commands!

Showing the first couple of terms with an ellipsis (...) is sufficient

First 2 terms ... last term

Car Dealership

A large auto dealership keeps track of sales made during each hour of the day. Let \( X \) = the number of cars sold during the first hour of business on a randomly selected Friday. Based on previous records, the probability distribution of \( X \) is as follows:

\[
\begin{array}{c|cccc}
\text{Cars sold:} & 0 & 1 & 2 & 3 \\
\text{Probability:} & 0.3 & 0.4 & 0.2 & 0.1 \\
\end{array}
\]

(a) compute and interpret the mean of \( X \)

\[
\mu_X = 0(0.3) + 1(0.4) + 2(0.2) + 3(0.1) = 1.1
\]

(b) compute and interpret the standard deviation of \( X \)

\[
\sigma_X = \sqrt{(0-1.1)^2(0.3) + (1-1.1)^2(0.4) + (2-1.1)^2(0.2) + (3-1.1)^2(0.1)} \\
= 0.94
\]
Homework

None tonight :-)

Partner Check!

Find and interpret the standard deviation of the Apgar scores.

<table>
<thead>
<tr>
<th>Value $x_i$:</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability $p_i$:</td>
<td>0.001</td>
<td>0.006</td>
<td>0.007</td>
<td>0.008</td>
<td>0.012</td>
<td>0.020</td>
<td>0.038</td>
<td>0.099</td>
<td>0.319</td>
<td>0.437</td>
<td>0.053</td>
</tr>
</tbody>
</table>

$M_x = 8.128$

Once you have your answer, check with your shoulder partner and compare. Make sure that they have adequate work shown!
Partner Check!

Find and interpret the standard deviation of the Apgar scores.

<table>
<thead>
<tr>
<th>Value $x_i$:</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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<td>0.099</td>
<td>0.319</td>
<td>0.437</td>
<td>0.053</td>
</tr>
</tbody>
</table>

Once you have your answer, check with your shoulder partner and compare. Make sure that they have adequate work shown!

$$\sigma_x = \sqrt{(0-8.128)^2(0.001) + (1-8.128)^2(0.006) + ... + (10-8.128)^2(0.053)}$$

$$\sigma_x = 1.44$$

The average distance the Apgar scores are from the mean is 1.44

6.1
Discrete & Continuous Random Variables

discoexamps

discovocab
Objectives

Today we will...

- Compute probabilities using the probability distribution of a continuous random variable

Continuous Random Variable

$X$

takes all values in an interval of numbers

the probability distribution of $X$ is defined by a density curve

the probability of any event is the area under the density curve and above/below the values of $X$ that make up the event
Continuous Random Variable

shoe size vs. foot length

How many possible foot lengths are there?

How can we graph the distribution of foot lengths?

Discrete or Continuous

number of hurdles cleanly jumped over

amount of time to run 110 meter hurdles

number of birthdays a student has had

student’s age
How do we find probabilities for continuous random variables?

Do you remember how to find area under a curve?

What calculator function do you use? \( \text{normalcdf/inVNorm} \)

How do you put in the value at the tail end? \( 1E99, 1E99 \)

Which 4 steps must you show? state plan do conclude

For a continuous random variable \( X \), how are \( P(X<a) \) and \( P(X\leq a) \) related

Same!

The probability of an individual outcome is 0

This is because there are infinite many outcomes and the probability is \( 1/\infty \)
Weights of 3 year old females

The weights of three-year-old females closely follow a Normal distribution with a mean of $\mu = 30.7$ pounds and a standard deviation of $\sigma = 3.6$ pounds. Randomly choose one three-year-old female and call her weight $X$.

(a) Find the probability that the randomly selected three-year-old female weighs at least 30 pounds.
(b) Find $P(25 < X < 35)$
(c) If $P(X < k) = 0.8$, find the value of $k$.

Weights of 3 year old females

(a) Find the probability that the randomly selected three-year-old female weighs at least 30 pounds.

STATE: We want to know the probability that a randomly chosen female is at least 30 lbs. The distribution is normal with $N(30.7, 3.6)$.

PLAN: We want to find the shaded area.

DO: $\text{normalcdf}(30, 1 \times 10^9, 30.7, 3.6) = 0.58$

min, max, mean, sd

CONCLUDE: The probability that a randomly selected 3-year-old female is at least 30 lbs is 0.58
Weights of 3 year old females

(b) Find the probability that a randomly selected three-year-old female weighs between 25 and 35 pounds.

STATE: We want to know the probability that a randomly chosen female is between 25 and 35 lbs. The distribution is normal with N(30.7,3.6).

PLAN: We want to find the shaded area.

DO: normalcdf(25,35,30.7,3.6) = .83

CONCLUDE: The probability that a randomly selected 3-year-old female is between 25 and 35 lbs is .83

---

Weights of 3 year old females

(c) If P(X<k) = 0.8, find the value of k.

STATE: We want to know the value of k such that 80% of weights are less than k. The distribution is normal with N(30.7,3.6).

PLAN: We want to find k that corresponds to an area below of .8.

DO: invNorm(.8,30.7,3.6) = 33.73 lbs

CONCLUDE: A 3 year old female who weights 33.73 pounds will have 80% of other females weights below her.
Homework

6.1 (day 3)