

# 11.2 Inference for Relationships (Day 1)

vocab

examples

## Objectives

## Difference Between

### 11.1 & 11.2

comparing distribution in two or more categorical variables/treatments vs.  
comparing distribution in just 1 category/treatment

one way table vs. two way table

ex/ comparing color distributions in regular m&ms and peanut m&ms

### Wine & Music

Wine	Music			Total
	None	French	Italian	
French	30	39	30	<b>99</b>
Italian	11	1	19	<b>31</b>
Other	43	35	35	<b>113</b>
<b>Total</b>	<b>84</b>	<b>75</b>	<b>84</b>	<b>243</b>

- Calculate the conditional distribution (in proportions) of the type of wine sold for each treatment.
- Make an appropriate graph for comparing the conditional distributions in part (a).
- Are the distributions of wine purchases under the three music treatments similar or different? Give appropriate evidence from parts (a) and (b) to support your answer.

### Wine & Music

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- (a) Calculate the conditional distribution (in proportions) of the type of wine sold for each treatment.
- (b) Make an appropriate graph for comparing the conditional distributions in part (a).
- (c) Are the distributions of wine purchases under the three music treatments similar or different? Give appropriate evidence from parts (a) and (b) to support your answer.



## Two explanations for differences:

The music that is playing truly influences a person's choice of wine

The differences are just due to chance alone

## Null & Alternative Hypothesis

$H_0$ : There is *no difference* in the distributions of (category 1) when (category 2) occurs

$H_a$ : There is a difference in the distributions of (category 1) based on (category 2)

\*note: this is called *many-sided*

## Multiple Comparisons

It is problematic to do many comparisons at once with an overall measure of confidence in all our conclusions. To deal with this conduct:

1. An overall test to see if there is good evidence of any differences among the parameters that we want to compare.
2. A detailed *follow-up analysis* to decide which of the parameters differ and to estimate how large the differences are.

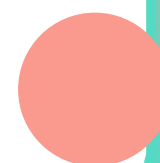
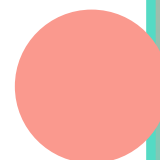
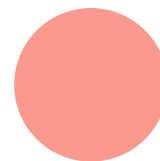


## Expected Counts

$$\frac{(\text{row total}) \times (\text{column total})}{\text{table total}}$$

don't round!

no formula :-)



## Wine & Music

Wine	Music			Total
	None	French	Italian	
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Find the expected counts

	None	French	Italian
french	39.22	30.56	31.22
italian	10.72	9.57	10.72
other	39.06	34.86	39.06

## Chi-Square Statistic

$$\chi^2 = \sum \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$$

Must do for all cells of the categories (except totals)

On formula sheet!

## Wine & Music

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Calculate Chi -Sqaure

Handwritten calculation of Chi-Square statistic:

$$\chi^2 = \frac{(30 - 39.22)^2}{39.22} + \frac{(11 - 10.72)^2}{10.72} + \frac{(43 - 39.00)^2}{39.00} + \frac{(39 - 30.56)^2}{30.56} + \frac{(1 - 9.57)^2}{9.57} + \frac{(35 - 34.80)^2}{34.80} + \frac{(30 - 31.22)^2}{31.22} + \frac{(19 - 10.72)^2}{10.72} + \frac{(35 - 39.06)^2}{39.06} = 18.27$$

The final result is boxed as 18.27.

## Degrees of Freedom

df = (number of rows - 1)(number of columns - 1)

$$(3-1)(3-1)$$

4

p-value = .00109

## BELLWORK

Mars Candy Company Claims:

Orange 20%, Red 13%, Yellow 14%, Green 16%, Blue 24%, Brown 13%

COLOR	OBSERVED	
Orange	290	237.6
Red	64	159.44
Yellow	143	166.32
Green	270	190.08
Blue	247	285.12
Brown	174	159.44
<b>TOTAL</b>	1188	

Is there convincing evidence that our class distribution of M&Ms differs from the claimed proportions?

$\chi^2$  GOF Test  
 using observed & expected  
 values & df = 5  
 $\chi^2 = 108.93$  p-value  $\approx 0$

## 11.2 Inference for Relationships (Day 2)

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## Conditions for Chi-Square Test

Random

Large Sample Size: All of the expected counts are at least 5

Independent: 10% Condition

## St. John's Wort

An article in the *Journal of the American Medical Association* (vol. 287, no. 14, April 10, 2002) reports the results of a study designed to see if the herb Saint-John's-wort is effective in treating moderately severe cases of depression. The study involved 338 subjects who were being treated for major depression. The subjects were randomly assigned to receive one of three treatments—Saint-John's-wort, Zoloft (a prescription drug), or a placebo—for an eight-week period. The table below summarizes the results of the experiment. Conduct and carry out a Chi-Square test to determine if there is convincing evidence that the treatment influences the response on depression.

	Saint-John's-wort	Zoloft	Placebo	Total
Full response	27	27	37	91
Partial response	16	26	13	55
No response	70	56	66	192
Total	113	109	116	338

## St. John's Wart

State:  $H_0$ : There is no difference in the distribution of the response of the patient based on the treatment that they receive

$H_a$ : There is a difference in the distribution of the response of the patient based on the treatment that they receive.

	Saint-John's-wort	Zoloft	Placebo	Total
Full response	27	27	37	91
Partial response	16	26	13	55
No response	70	56	66	192
Total	113	109	116	338

$\alpha = .05$

### St. John's Wart

PLAN: Random-yes, independent - assuming at least 3380 patients. Large sample size -

expected:

	SJW	Z	P'
full	30.42	29.35	31.23
part	18.39	17.74	18.08
no	64.19	61.92	65.89

all are  $\geq 5$   
We will use a  $\chi^2$  test

	Saint-John's-wort	Zoloft	Placebo	Total
Full response	27	27	37	91
Partial response	16	26	13	55
No response	70	56	66	192
Total	113	109	116	338

### St. John's Wart

DO:  $\chi^2$  test using observed & expected values

$$\chi^2 = 8.72 \quad p\text{-value} = .069$$

$$df = 4$$

	Saint-John's-wort	Zoloft	Placebo	Total
Full response	27	27	37	91
Partial response	16	26	13	55
No response	70	56	66	192
Total	113	109	116	338

## St. John's Wart

Conclude:  $.0092 > .05 \rightarrow$  We fail to reject  $H_0 \rightarrow$   
 We can't conclude that there is a difference  
 in the distribution of response based  
 on treatment. The results are NOT  
 statistically sig. at the 5% level.

	Saint-John's-wort	Zoloft	Placebo	Total
Full response	27	27	37	91
Partial response	16	26	13	55
No response	70	56	66	192
Total	113	109	116	338

## BirthDAYS

Has modern technology changed the distribution of birthdays? With more babies being delivered by planned c-section, a statistics class hypothesized that the day-of-the-week distribution for births would be different for people born after 1993 compared to people born before 1980. To investigate, they selected a random sample of people from each both age categories and recorded the day of the week on which they were born. The results are shown in the table. Is there convincing evidence that the distribution of birth days has changed?

	Before 1980	After 1993	
Sunday	12	9	21
Monday	12	11	23
Tuesday	14	11	25
Wednesday	10	10	20
Thursday	6	17	23
Friday	9	9	18
Saturday	10	6	16
	73	73	146

## BirthDAYS

**STATE:**  $H_0$ : There is no difference in the distribution of birthdays based on birth year

$H_a$ : There is a difference in the distribution of birthdays based on birth year

$\alpha = .05$

	Before 1980	After 1993	
Sunday	12	9	<b>21</b>
Monday	12	11	<b>23</b>
Tuesday	14	11	<b>25</b>
Wednesday	10	10	<b>20</b>
Thursday	6	17	<b>23</b>
Friday	9	9	<b>18</b>
Saturday	10	6	<b>16</b>
	<b>73</b>	<b>73</b>	<b>146</b>

## BirthDAYS

**PLAN:** Random - yes    Independent - Assuming at least 1460 births

Large Sample Size: The expected counts are all at least 5

We will use a Chi-Square Test

	Before 1980	After 1993	
Sunday	12	9	<b>21</b>
Monday	12	11	<b>23</b>
Tuesday	14	11	<b>25</b>
Wednesday	10	10	<b>20</b>
Thursday	6	17	<b>23</b>
Friday	9	9	<b>18</b>
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	<b>73</b>	<b>73</b>	<b>146</b>

	Before 1980	After 1993
Sunday	10.5	10.5
Monday	11.5	11.5
Tuesday	12.5	12.5
Wednesday	10	10
Thursday	11.5	11.5
Friday	9	9
Saturday	8	8



## BirthDAYS

**DO:**  $\chi^2$  TEST using the given observed and above expected values

df = 6

$\chi^2 = 7.09$       p-value = .312

	Before 1980	After 1993	
Sunday	12	9	<b>21</b>
Monday	12	11	<b>23</b>
Tuesday	14	11	<b>25</b>
Wednesday	10	10	<b>20</b>
Thursday	6	17	<b>23</b>
Friday	9	9	<b>18</b>
Saturday	10	6	<b>16</b>
	<b>73</b>	<b>73</b>	<b>146</b>

## BirthDAYS

**CONCLUDE:**  $.312 \geq .05$  --> We fail to reject the  $H_0$  --> We cannot conclude that there is a difference in the distribution of day of birth based on birth year. The results are not statistically significant at the 5% level.

	Before 1980	After 1993	
Sunday	12	9	<b>21</b>
Monday	12	11	<b>23</b>
Tuesday	14	11	<b>25</b>
Wednesday	10	10	<b>20</b>
Thursday	6	17	<b>23</b>
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## Follow Up

## Computer Output

### **Chi-Square Test: None, Franch, Italian**

Expected counts are printed below observed counts

Chi-Square contributions are printed below expected counts

	None	French	Italian	Total
1	30	39	30	99
	34.22	30.56	34.22	
	0.521	2.334	0.521	
2	11	1	19	31
	10.72	9.57	10.72	
	0.008	7.672	6.404	
3	43	35	35	113
	39.06	34.88	39.06	
	0.397	0.000	0.422	
Total	84	75	84	243

Chi-Sq = 18.279, DF = 4, P-Value = 0.001

## Ibuprofen vs. Acetaminophen

In a study reported by the *Annals of Emergency Medicine* (March 2009), researchers conducted a randomized, double-blind clinical trial to compare the effects of ibuprofen and acetaminophen plus codeine as a pain reliever for children recovering from arm fractures. There were many response variables recorded, including the presence of any adverse effect, such as nausea, dizziness, and drowsiness. Here are the results:

	Ibuprofen	Acetaminophen plus codeine	Total
Adverse effects	36	57	93
No adverse effects	86	55	141
Total	122	112	234

(a) Calculate the chi-square statistic and  $P$ -value.

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No adverse effects	86	55	141
Total	122	112	234

(a) Calculate the chi-square statistic and  $P$ -value.

$$X^2 = 11.15$$

$$p\text{-value} = .00084$$

$$df = 1$$

## Ibuprofen vs. Acetaminophen

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(b) Show that the results of a two-sample z test for a difference in proportions are equivalent.

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(b) Show that the results of a two-sample z test for a difference in proportions are equivalent.

2-PropZTest

x1: 36      n1: 122

x2: 57      n2: 112

p1 ≠ p2

p-value = .00084

## Chi-Square vs. Two-Sample

